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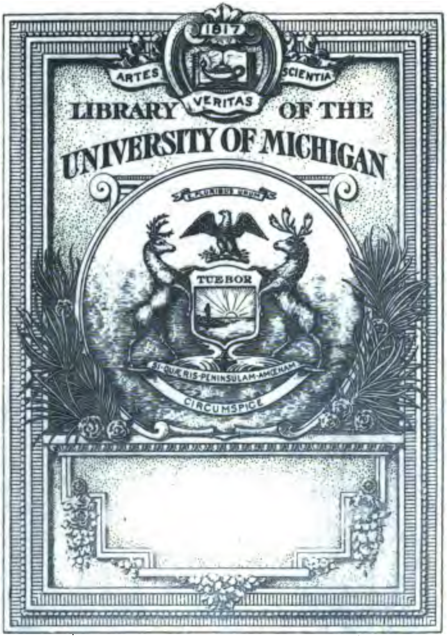

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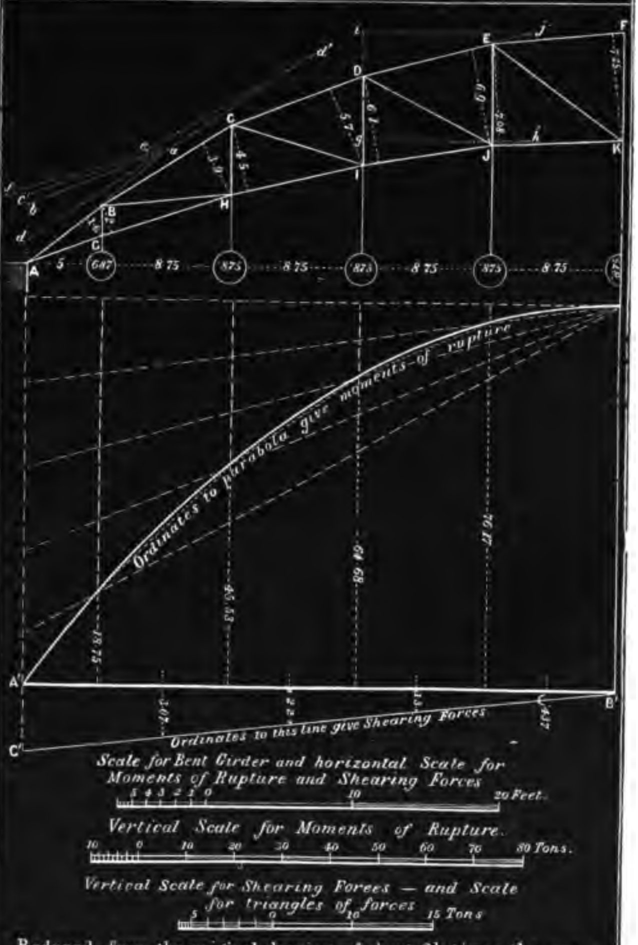
PLATE I.

METHOD OF APPLYING DIAGRAMS OF STRAINS.


A
B
C
D
E
F
G
H
I
J
K
a
b
c
c'
d
d'
f
g
h
i
j
5
687
8.75
875
3.9
4.5
5.7
6.1
6.9
7.08
7.25
Ordinates to parabola give moments of rupture
18.75
46.53
A'
B'
C'
3.07
.437
Ordinates to this line give Shearing Forces
Scale for Bent Girder and horizontal Scale for Moments of Rupture and Shearing Forces
5 4 3 2 1 0
10
20 Feet.
Vertical Scale for Moments of Rupture.
10
0
10
20
30
40
50
60
70
80 Tons.
Vertical Scale for Shearing Forces — and Scale for triangles of forces
5
0
10
15 Tons

Reduced from the original drawing of about 2½ times the sizes.

UNIV. OF MICH.

A

HANDY-BOOK

FOR THE CALCULATION OF

STRAINS IN GIRDERS

AND SIMILAR STRUCTURES, AND THEIR STRENGTH;

CONSISTING OF

FORMULÆ AND CORRESPONDING DIAGRAMS,

WITH

NUMEROUS DETAILS FOR PRACTICAL APPLICATION,

ETC., ETC.

BY WILLIAM HUMBER, ASSOC. INST. C.E.

AUTHOR OF "THE WATER-SUPPLY OF CITIES AND TOWNS," "A PRACTICAL TREATISE ON CAST AND WROUGHT IRON BRIDGE CONSTRUCTION," ETC., ETC.

Fifth Edition.

LONDON:
CROSBY LOCKWOOD AND SON,
7, STATIONERS' HALL COURT, LUDGATE HILL.
1891.

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Gift
Dean M.E. Cooley
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PREFACE.

NOTWITHSTANDING that the subject of Strains has been ably treated of again and again, it is difficult in submitting this little work to the public to avoid the almost stereotyped expression that "the design has been to supply a want long felt in the profession;" for the numerous volumes which have appeared on the subject have not, principally on account of their elaborate investigations, been calculated to afford that ready assistance which in the ordinary run of office and other work is being continually needed, while on the other hand, most of the general Engineering Pocket-books, not having been able to afford sufficient space to do justice to the subject, have been compelled to leave its treatment incomplete. It is hoped, therefore, that by devoting a small work, in a handy form, entirely to Bridge and Girder Calculations, without giving more than is absolutely necessary for the complete solution of practical problems, both the above obstacles to quick and satisfactory manipulation may be overcome.

One of the chief features of the present work is the extensive application of simply constructed *diagrams* to the calculation of the strains on bridges and girders, the advantages of which as a system, most undeniably far outweigh its disadvantages. The parabola (anything but a troublesome figure to draw) and a few right lines are all that are required.

There is, again, a more general application of the Moments of Rupture, and Shearing Forces, to open-webbed girders of all kinds, than has hitherto been attempted.

It was originally intended to divide the *whole* work into three sections or chapters, similar to those actually adopted only for the middle portion (pp. 24 to 60), which chapters should correspond with the various processes in the design of a bridge, thus making the very arrangement of the work a general guide. But it was afterwards deemed advisable, as will be seen, to place the Moments of Rupture, and Shearing Forces, by themselves at the commencement, as a basis upon which the remainder is principally founded.

In the following pages will be found, almost necessarily, many omissions, but care has been taken as far as possible to avoid inaccuracies. It will be observed that attention has been paid to the arrangement of the matter in different types, so as to facilitate as far as possible the manipulation of the contents. The work is not advanced with the pretensions of a treatise, as no *investigations* whatever are given, but merely their *results;* and these, it is hoped, in an intelligible and practical form, suited to the wants of the Engineer, Architect, Draughtsman, or Builder

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FORMULÆ AND DIAGRAMS

FOR THE

CALCULATION OF GIRDERS, &c.

STRAINS IN BEAMS.

1. **THE STABILITY OF A LOADED BEAM** or girder is founded on the equality that must always exist between the resultants of all the various external forces tending to cause its rupture, and the sum of the molecular reactions which resist the same. The former may be resolved—(1) *horizontally* into strains, depending for their value upon what are known as *Moments of Rupture*, or *Bending Moments*, tending to cause the failure of the beam, by tearing asunder its fibres in one part and crushing them together in another (**4**) : and (2) *vertically* into what are known as *Shearing Forces*, due to the transmission of the vertical pressure of the load to the points of support, and tending to cause contiguous vertical sections in the beam to slide over each other (**171**). The values of the molecular reactions are the *Moments of Resistance*, for which see (**204—214**).

MOMENTS OF RUPTURE.

2. Abbreviations adopted in the Formulæ.

M_x = moment of rupture at any point (x).
M_A, M_B = ,, at points of support (A, B).
M_C = ,, at centre of span.
l = length of clear span = distance between supports in a whole beam = distance between W and support in a semi-beam loaded with W. Where used in any other way explanation will be made.
x = horizontal distance between the left abutment (except where otherwise stated) and the point at which M is to be found.
W = concentrated load at any point (**61**).
w = distributed stationary or dead load per unit of length.
w' = ,, moving or live ,, ,, (**61**).
Def. = maximum deflection (**221**) for a beam of uniform section (**223**).
I = moment of inertia of the section of the beam (for value see **204—214**).
E = modulus of elasticity (**173**) ; for value see **231.**
Other abbreviations will be explained as they occur.

3. NOTE. l, x, and other horizontal distances when occurring in the same case, must be all of the same denomination ; and so also must w and w'.

4. NOTE.—The value of a formula being $\begin{pmatrix}\text{positive }(+)\\ \text{negative}(-)\end{pmatrix}$ shows that the action of the load makes, or tends to make the upper surface of the beam $\begin{pmatrix}\text{concave,}\\ \text{convex,}\end{pmatrix}$ and therefore compresses together the fibres in the $\begin{pmatrix}\text{upper}\\ \text{lower}\end{pmatrix}$ part, and stretches them in the $\begin{pmatrix}\text{lower}\\ \text{upper}\end{pmatrix}$ part.

5. In the DIAGRAMS, the ordinates (the vertical distances from the horizontal, or other lines) to the curves, &c., as shown thereon, correspond to the values of the formulæ accompanying them. If the diagram be drawn to scale in the manner directed, the Moments of Rupture may be obtained by *direct measurement.*

Fig. A.

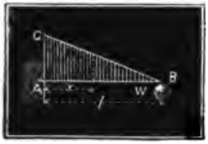
Fig. B.

The vertical lines, as in fig. A, correspond to positive, and those as in fig. B to negative values in the formulæ (4).

6. When a diagram is used in the calculation of the moments, it should be drawn immediately under, or over, and to the same horizontal scale as the outline sketch (53, III., and 59) of the girder itself, so that the ordinates in the diagram will correspond with the several points in the length of the girder to which they apply.

7. Semi-Beam Fixed at One End, and Loaded with a Concentrated Weight at the Other.

Fig. 1.



$$M_A = -W\,l \quad . \quad . \quad . \quad (4.)$$

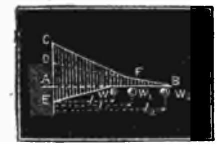
$$M_x = -W\,(l-x) \quad . \quad (4.)$$

$$\text{Def.} = \frac{W\,l^3}{3\,E\,I}.$$

DIAGRAM.—Let A B be the beam (5, 6). Draw A C = W *l*. Join C to B. Then the vertical distances between A B and C B will give the moments of rupture.

8. Semi-Beam Fixed at One End, and Loaded with several Concentrated Weights, W, W_1, W_2 (37).

Fig. 2.



$$M_A = -(W\,l + W_1\,l_1 + W_2\,l_2) \quad . \quad . \quad . \quad (4.)$$

$$M_x = -\{W(l-x) + W_1\,(l_1-x) + W_2\,(l_2-x)\} \quad (4.)$$

When $(l-x)$ or (l_1-x), or so on, is negative, it is to be omitted.

DIAGRAM.—Let A B be the beam (5, 6). Draw A D = $W_2\,l_2$, D C = $W_1\,l_1$, and A E = W *l*. Join D to B, E to W, and C to F, in the manner shown. Then the vertical distances between C F B and E W B will give the moments of rupture.

9. Semi-Beam Fixed at One End, and Loaded Uniformly its Entire Length (38).

$M = -\frac{w\,l^2}{2}$; *one-half that due to the same load* ($w\,l$) *concentrated at the end.*

$$M_x = -\frac{w}{2}(l-x)^2 \quad . \quad . \quad (4.)$$

$$\text{Def.} = \frac{w\,l^4}{8\,E\,I}.$$

Fig. 3.

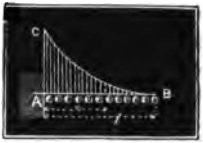


DIAGRAM.—Let A B be the beam (5, 6). Draw A C $= \frac{w\,l^2}{2}$. Draw the parabola C B, whose vertex is at B (232). Then the vertical distances between A B and C B will give the moments of rupture.

10. Semi-Beam Fixed at One End, with a Load Uniformly Distributed over Part of its Length.

Let z = the length of the load;—

$$M_A = -w\,z\left(l - \frac{z}{2}\right).$$

When x is less than or equal to $(l-z)$, then

$$M_x = -w\,z\left(l - \frac{z}{2} - x\right).$$

When x is greater than $(l-z)$, then

$$M_x = -\frac{w}{2}(l-x)^2 . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

Fig. 4.

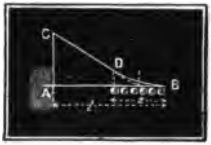


DIAGRAM.—Let A B be the beam (5, 6). At A erect A C $= w\,z\left(l - \frac{z}{2}\right)$. Join C to a point in A B at the mid length of the load z (as in the Fig.). Draw the semi-parabola D B (232) the same as for a beam of the length z, fully loaded (9). The vertical distances between A B and C D B will give the moments of rupture.

11. Semi-Beam Fixed at One End, and Loaded with a Uniformly Distributed Load, and also a Concentrated Load at its Extremity (39).

$$M_A = -\left(W + \frac{w\,l}{2}\right)l. \quad . \quad . \quad . \quad . \quad (4.)$$

$$M_x = -\left(W(l-x) + \frac{w}{2}(l-x)^2\right) \quad (4.)$$

$$\text{Def.} = \frac{l^3}{E\,I}\left(\frac{W}{3} + \frac{w\,l}{8}\right).$$

Fig. 5.

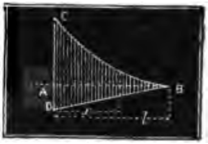
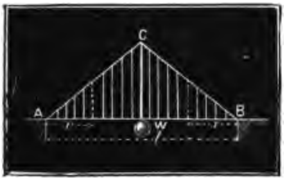


DIAGRAM.—Let A B be the beam (5, 6). Draw A C $= \frac{w\,l^2}{2}$, and A D $= W\,l$. Draw the parabola C B (as in 9) with its vertex at B (232). Then the vertical distances between D B and C B will give the moments of rupture. This is but a combination of (7) and (9).

12. Beam Supported at Both Ends, and Loaded at the Centre (40).

Fig. 6.



$$M_A = M_B = 0. \qquad M_C = \frac{W\,l}{4}.$$

$M_x = \frac{W\,x}{2}$ (x being measured from the nearer pier).

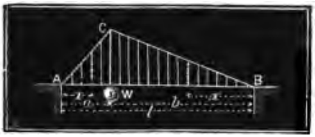
$$\text{Def.} = \frac{W\,l^3}{48\,E\,I}.$$

Diagram.—Let A B be the beam (5, 6). At mid-span erect C W $= \frac{W\,l}{4}$. Join C to A and B. Then the vertical distances between A B and A C, C B will give the moments of rupture.

13. Beam Supported at Both Ends, and Loaded with a Concentrated Weight at any Point (41).

The greatest moment will always be obtained at the point of application of the load, and $= \frac{W\,a\,b}{l}$.

Fig. 7.



$$M_A = M_B = 0.$$

Between A and W, $M_x = \frac{W\,x\,b}{l}$.

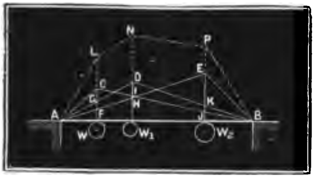
Between B and W, $M_x = \frac{W\,x\,a}{l}$.

Note particularly that x must be measured from the pier which is on the same side of W as x is.

Diagram.—Let A B be the beam (5, 6). At W erect W C $= \frac{W\,a\,b}{l}$. Join C to A and B. Then the vertical distances between A B and A C, C B will give the moments of rupture.

14. Beam Supported at Both Ends, and Loaded with any Number of Concentrated Weights at any Points.

Fig. 8.



The moment of rupture at any point produced by all the weights is the sum of the moments produced at that point by each of the weights separately (13 and 33A).

Let there be three weights, W, W_1, W_2, and let the segments into which they each divide the beam be respectively a, b; a_1, b_1; a_2, b_2; then supposing that x is taken between W_1 and W_2.

$$M_x = \frac{1}{l}\,(W\,a\,x + W_1\,a_1\,x + W_2\,b_2\,x),$$ having regard to the note in (13).

$$M_A = M_B = 0.$$

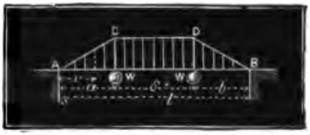
Diagram.—Let A B be the beam (5, 6). Draw A C B, A D B, and A E B as for three separate cases, by (13). Produce W C till W L = W F + W G + W C. Produce W_1 D till W_1 N = W_1 H + W_1 I + W_1 D; and so on for the weight W_2, making W_2 P = W_2 K + W_2 J + W_2E. Join A to L, L to N, N to P, and P to B. Then the ordinates from A B to the polygonal figure A L N P B will give the moments of rupture.

Note. If the weights be all equal, the verticals at the weights representing the moments produced there by those weights (as W C, W_1 D, W_2 E) will all be ordinates to a parabola (232) drawn as for (17).

The following cases (15, 16) are adaptations of the above scheme to conditions frequently met with in practice.

15. Beam Supported at Both Ends, and Loaded with Two Equal Weights placed Equidistant from the Centre.*

Fig. 9.



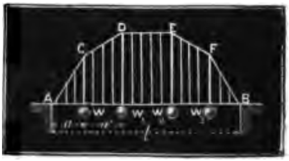
The moment for any point between the weights is a constant quantity $= \dfrac{W\,(l - \delta)}{2} = W\,a = W\,b.$

Between the weights and the supports $M_x = W_x$. $M_A = M_B = 0.$

Diagram.—Let A B be the beam (5, 6). At the weights erect W C and W D each equal to (W a). Join A to C, C to D, and D to B. Then the vertical distances between A B and A C D B will give the moments of rupture.

16. Beam Supported at Both Ends, and Loaded with Four Equal Weights symmetrically disposed about the Centre.†

Fig. 10



M at W_1 = M at W_2 = W $(2\,a + a')$; constant from W_1 to W_2.

M at W = M at W_3 = 2 W a.

$$M_A = M_B = 0.$$

Diagram.—Let A B be the beam (5, 6). At W and W_3 erect W C and W_3 F, each equal to (2 Wa); and at W_1 and W_2 erect W_1 D and W_2 E, each equal to W$(2a + a')$. Join A to C, C to D, D to E, E to F, and F to B. Then the vertical distances between A B and A C D E F B will give the moments of rupture.

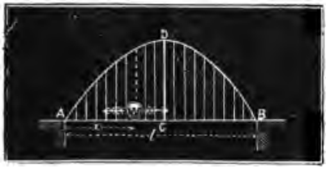
---

* As in the case of a cross girder carrying a single line of railway.

† Case of a cross girder carrying a double line of railway.

17. Beam Supported at Both Ends, Loaded with a Concentrated Rolling Weight (42).

Fig. 11.



The maximum moment at any point—

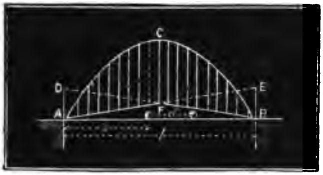
$$M_x = \frac{W\,x\,(l-x)}{l}.$$

$$M_A = M_B = 0.$$

Diagram.—Let A B be the beam (5, 6). Draw the parabola A D B (232), whose ordinate at centre (C D) $= \frac{W\,l}{4}$. Then the vertical distances between A B and the parabola A D B will give the maximum moments of rupture.

18. Beam Supported at Both Ends, and Loaded with Two Weights moving simultaneously in either direction over the Beam.*

Fig. 12.



Let w and w_1 be the two weights. The value of the maximum moment produced at any point is

$$M_x = \frac{x}{l}\left\{(w+w_1)\,(l-x) - w_1\,\delta\right\}$$

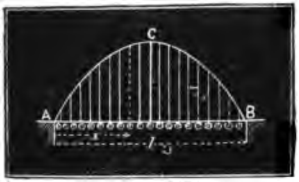
x being measured from the nearest pier. The position of w causing the greatest moment is when $x = \frac{l}{2} \pm \frac{w_1\,\delta}{2(w+w_1)}$. Or if the two weights be equal; when $x = \frac{l}{2} \pm \frac{\delta}{4}$.

$$M_A = M_B = 0.$$

Diagram.—Let A B be the beam (5, 6). Draw the parabola A C B (232), whose ordinate at centre $= \frac{(w + w_1)\,l}{4}$. At A and B erect A D and B E $=$ $w_1\,\delta$. Join A to E and D to B. Then the vertical distances between A F B and the parabola A C B will give the maximum moments of rupture.

19. Beam Supported at Both Ends, and Loaded uniformly its entire length (44).

Fig. 13.



$$M_x = \frac{w\,x}{2}\,(l-x). \qquad M_C = \frac{w\,l^2}{8}.$$

$$M_A = M_B = 0.$$

$$\text{Def.} = \frac{5\,w\,l^4}{384\,E\,I};\text{—}$$

one half the moment at centre, and $\frac{5}{8}$ the deflection produced by the same load concentrated at the centre.

Diagram.—Let A B be the beam (5, 6). On A B draw the parabola

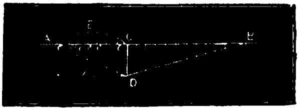
---

* As in the coupled driving wheels of a locomotive.

A C B (232) whose ordinate at centre $= \frac{w\,l^2}{8}$. Then the vertical distances between A B and the parabola A C B will give the moments of rupture.

20. Beam supported at Both Ends, subject to a Load uniformly Distributed over a certain Length from one Support (45).

Fig. 14.



Let z equal the length of the load, and let x be measured from the abutment from which the load advances.

The greatest moment produced by a given length of load will be at its extremity, or when $x = z$, that is, provided the load does not extend beyond the centre of the span; for should it depass that point, the greatest strain will remain constant in position at the midspan, increasing in intensity until the load completely covers the span.

When $x = z$, or more than z, $M_x = \frac{w\,z^2\,(l - x)}{2\,l}$.

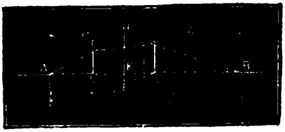
When x is less than z, $M_x = \frac{w\,x}{2}\left\{\frac{z\,(2\,l - z)}{l} - x\right\}$.

$$M_A = M_B = 0.$$

DIAGRAM.—Let A B be the beam (5, 6). At the extremity (C) of the load draw $C\,D = \frac{w\,z^2 (l - z)}{2\,l}$. Join A to D, and B to D. Draw the parabola A E C (232), whose ordinate at its centre $= \frac{w\,z^2}{8}$; the same as if for a uniform load on a beam A C, supported at A and C. (19). Then the vertical distances between A D B and A E C B will give the moments of rupture.

21. Beam Supported at Both Ends, subject to a Load uniformly Distributed over a certain Length not Extending to either Support.

Fig. 15.



Let z = the length of the load.
Let v = the distance from the load to the left support; and let x be measured from the same support.

When $x = v$, or less than v,

$$M_x = \frac{w\,z\,x}{l}\left(l - v - \frac{z}{2}\right).$$

When x is equal to or or greater than $(v + z)$,

$$M_x = \frac{w\,z}{l}\left(v + \frac{z}{2}\right)(l - x).$$

When x is greater than v, and less than $(v + z)$,

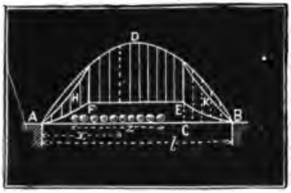
$$M_x = \frac{w\,x}{2}\left\{2\,z + 2\,v - x - \frac{z\,(z + 2\,v)}{l}\right\} - \frac{w\,v^2}{2}.$$

$$M_A = M_B = 0$$

DIAGRAM.—Let A B be the beam (**5**, **6**). And let the load extend over the length z. At K, the centre of the load, erect $KH = \frac{wzab}{l}$. Join H to A and H to B. At C and D, the extremities of the load, draw perpendiculars to A B, intersecting H A and H B in F and G respectively. Join F to G. On C D, draw the parabola C E D (**232**), whose ordinate at centre is equal to $\frac{wz^2}{8}$, the same as for a distributed load on a beam of the length z, and supported at C and D (**19**). Then the vertical distances between A C E D B and A F G B will give the moments of rupture.

22. Beam Supported at Both Ends, subject to a Rolling Distributed Load of a Length less than that of the Beam.

Fig. 16.



Let z = the length of the load.

Maximum moment at any point,

$$M_x = \frac{wzx(l - x)}{l} - \frac{wz^2}{8}.$$

This will cease to give the correct value when x approaches nearer either support than $\frac{z}{2}$, beyond which limits z in the formula must be taken equal to $2x$.

DIAGRAM.—Let A B be the beam (**5**, **6**). On A B draw the parabola A D B (**232**), whose ordinate at centre $= \frac{wzl}{4}$. Make $CE = \frac{wz^2}{8}$. Through E draw E F parallel to A B. Make the horizontal distance of F and E from A and B respectively (as C B) $= \frac{z}{2}$. The diagram will be accurate from F to E. But for the construction beyond those limits proceed as follows. Draw the semi-parabola E B (**232**) whose vertex is at B. Divide C B into a number of parts n (say five), and at the divisions draw verticals. At the first division from B take $\frac{1}{n}$ ($= \frac{1}{5}$) of the vertical distance between the parabolic arcs E B, and D B measuring from E B. At the second division take $\frac{2}{n}$ and so on, counting the divisions from the abutment, and measuring from the smaller (lower) parabola, E B. The points thus obtained will enable the curve, as shown in the diagram, to be traced through. Repeat the same operation on the opposite end of the beam. And then the vertical distances between A H D K B and A F E B will give the moments of rupture.

23. Points of Contrary Flexure, or of inflexion, or of "no-curvature," as they are sometimes called, are points at which the upper and lower surfaces change from convexity to concavity (**4**), and *vice versâ* (see fig. 23). At these points, as there is no curvature there is no moment of rupture, for the moments of rupture are the intensities of the curving or bending forces (**1**).

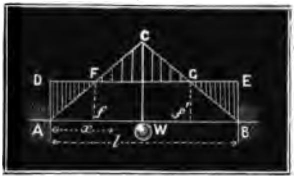
24. Beam of Equal and Uniform Section, or Beam of Uniform Strength (165), Fixed horizontally at the Ends, and Loaded at the Centre. (See 30.)

The length ff' *is identical with* (12), *and the parts* A f *and* B f' *with* (7).

$$M_c = \frac{W\,l}{8}. \qquad M_x = \frac{W}{2}\left(x - \frac{l}{4}\right).$$

$$M_A = M_B = -\frac{W\,l}{8}. \quad . \quad . \quad . \quad (4.)$$

Fig. 17.



Where $M_x = 0$ there are *points of contrary flexure*, distant from their respective piers by $\frac{1}{4}\,l$.

Diagram.—Let A B be the beam (5, 6). At mid-span erect C W $= \frac{W\,l}{4}$. At A and B erect A D and B E, each $= \frac{W\,l}{8}$. Join D to E, and C to A and B. Then the vertical distances between D E and A C B will give the moments of rupture.

The *points of contrary flexure* (23) are at the intersection of A C and C B, with D E.

25. Beam of Equal and Uniform Section, Fixed horizontally at Both Ends, and Loaded uniformly its entire Length (47). (See 30.)

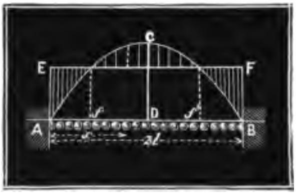
The length ff' *is identical with* (19), *and the parts* A f *and* B f' *with* (11), the concentrated load at the extremity of each semi-beam A f, B f' being equal to half the distributed load on ff'.

$$M_c = \frac{w\,l^2}{24}.$$

$$M_x = \frac{w\,x}{2}(l - x) - \frac{w\,l^2}{12}.$$

$$M_A = M_B = -\frac{w\,l^2}{12}. \quad . \quad . \quad (4.)$$

Fig. 18.



Where $M_x = 0$, there are the *points of contrary flexure* (23), distant from A and B respectively by $\cdot 211\,l$.

Def. $= \frac{5\,w\,l^4}{1536\,E\,I}$ = *one quarter that of the same beam if not fixed at the ends.*

Diagram.—Let A B be the beam (5, 6). On A B draw the parabola A C B (232), whose ordinate at centre C D $= \frac{w\,l^2}{8}$. At A and B erect

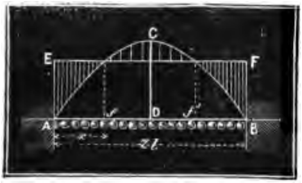
A E and B F respectively, each equal to $\frac{w\,l^2}{12}$. Join E to F. Then the vertical distances between E F and A C B will give the moments of rupture.

The *points of contrary flexure* (23), are at the intersection of A C B with E F.

26. Beam of Uniform Strength (165), Fixed horizontally at Both Ends, and Loaded uniformly its entire Length (47). (See 30.)

The length $f\,f'$ $(= \frac{1}{2}\,l)$ *is identical with* (19), *and the parts* A f *and* B f' *with* (11); the concentrated load at the extremity of each being equal to $\frac{w\,l}{4}$ = half the load on ff'.

Fig. 19.



$$M_C = \frac{w\,l^2}{32}.$$

$$M_x = \frac{w\,x}{2}\,(l - x) - \frac{3\,w\,l^2}{32} \quad (4.)$$

$$M_A = M_B = -\frac{3\,w\,l^2}{32} \quad (4.)$$

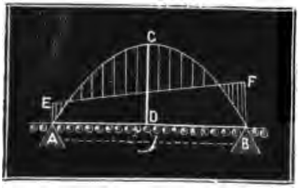
Where $M_x = 0$, there are the *points of contrary flexure* (23).

$$A\,f = B\,f' = \frac{l}{4}.$$

Diagram.—Let A B be the beam (5, 6). On A B draw the parabola A C B (232), whose ordinate at centre (C D) = $\frac{w\,l^2}{8}$. At A and B erect A E and B F respectively, each = $\frac{3\,w\,l^2}{32}$. Join E to F. Then the vertical distances between E F and A C B will give the moments of rupture. The *points of contrary flexure* (23) are at the intersection of A C B with E F.

27. Beam of Uniform and Equal Section, resting on Two Supports and uniformly Loaded, the Extremities being Subjected to two known Moments of Rupture (M_A, M_B) acting in a Contrary Direction to those due to the Load (46).

Fig. 20.



$$M_x = \frac{w\,x}{2}\,(l - x) - M_A + \frac{x}{l}\,(M_A - M_B).$$

Where $M_x = 0$, there are the *points of contrary flexure.*

Diagram.—Let A B be the beam (5, 6). On A B draw the parabola A C B (232), whose ordinate at centre C D = $\frac{w\,l^2}{8}$. At A and B erect A E and B F respectively, making $AE = M_A$, and $BF = M_B$. Join E to F. Then the vertical distances between E F and A C B will give the moments of rupture. Where E F intersects A C B, there will be the *points of contrary flexure* (23).

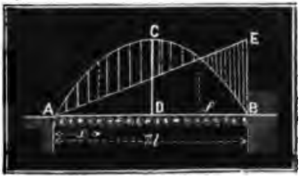
28. Beam of Uniform and Equal Section, supported at One End (A), and Fixed horizontally at the Other (B)*, uniformly Loaded over its entire Length (49).

This case is identical with the length f B *of* (25).

$$M = \frac{wx}{2}(l - x) - \frac{wlx}{8}$$ (x being measured from the unfixed end).

$$M_A = 0. \qquad M_B = -\frac{wl^2}{8}\ . \quad (4.)$$

Fig. 21.



The *point of contrary flexure* (23) is where $M_x = 0$.

$$Af = \frac{3l}{4}.† \qquad M \text{ at midway between A and } f = \frac{9wl^2}{128}.$$

DIAGRAM.—Let A B be the beam (5, 6). On A B draw the parabola A C B (232), whose ordinate at centre $CD = \frac{wl^2}{8}$. At B, the fixed end of the beam, erect $BE = \frac{wl^2}{8} = CD$. Join A to E. Then the vertical distances between A E and A C B will give the moments of rupture. Where A E intersects C B, there will be the *point of contrary flexure* (23).

29. Beam of Uniform Strength (165), supported at One End (A) and Fixed horizontally at the other (B)*, uniformly Loaded over its entire Length (48).

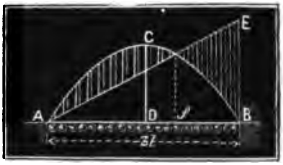
This case is identical with the length f B *of* (26).

$$M_x = \frac{wx}{2}(l - x) - \frac{wlx}{6},$$ (x being measured from the unfixed end).

$$M_A = 0. \qquad M_B = -\frac{wl^2}{6}\ . \quad (4)$$

M at midway between A and $f = \frac{wl^2}{18}$.

Fig. 22.



The *point of contrary flexure* (23) is where $M_x = 0$. $\qquad Af = \frac{2}{3}l.$

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* See 30.

† The values in this case are slightly erroneous. The correct value for A *f* is ·733 *l*.

DIAGRAM.—Let A B be the beam (5, 6). On A B draw the parabola A C B (232), whose ordinate at centre (C D) $= \frac{wl^2}{8}$. At B, the fixed end of the girder, erect B E = $M_B = \frac{wl^2}{6}$. Join A to E. Then the vertical distances between A E and A C B will give the moments of rupture. Where A E intersects A C B, there is the *point of contrary flexure* (23).

30. Beam continuous for two or more Rightly Proportioned * Spans, subject to a Stationary Load, Uniformly Distributed over its entire Length (50).

All such cases may be regarded as combinations of some of the cases previously given (24—29). For if, in any of the latter, the beam, instead of being fixed at one or both ends, be continued over a support where originally fixed, and subjected to the action of a load, which shall produce at the point of support a moment equal to that produced there by the first load when the beam was fixed at that support, then the moments in the original length of the beam will remain as they were, and will not be affected by the substitution of the continuation for the fixing.

Fig. 23.

Let A F be a beam continuous over a number of equidistant piers, B, C, &c.

If the beam be of uniform strength (165), the outer spans (A B and E F) should be $\frac{3}{4}$ the length of the others. If of uniform section, the outer spans should be ·789 the length of the others. (See **113 A.**)

The end spans may be regarded as identical with (28) or (29), and the remaining spans with (24, 25, or 26), so that the moments of rupture may be obtained from the formulæ or diagrams there given.

31. Continuous Beams, not of Uniform Section, subject to Varying Loads.

It would, perhaps, be impossible to give mathematically accurate formulæ for the moments of rupture in continuous beams, with moving loads, that would be worth anything for practical application. A well-known author † has even pronounced the case "too complicated for investigation."

The following approximations, however, may be relied on for *safety without extravagance.*

32. Beam of Uniform Strength (165) for the Maximum Strains,

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* By rightly proportioned is meant, proportioned so that if the beam were fixed on any one of the piers instead of continuous over it, the moments produced there by the two adjacent loaded spans would be equal. Then *if this condition be observed* the case will include beams not uniformly loaded throughout.

† J. H. Latham, Esq., M.A.—"*Iron Bridges.*"

continuous over one Pier, forming two Equal Spans, subject to a Fixed Load Uniformly Distributed, and also to a Moving Load (51). (See **113 A.**)

The greatest moment over the pier will be produced when both spans are fully loaded. Each span will then nearly correspond to (**29**).

The greatest positive (4) *moment* will obtain in the span fully loaded, when the other span bears only the fixed load.

Let w = fixed load per unite of length.
w' = varying ,, ,,
and let x be always measured from an abutment, and *not from the pier. Then,*

Maxm. *positive* (4) moment, $M_x = \frac{x\,(w + w')\,(l - x)}{2} - \frac{(2\,w + w')\,x\,l}{12}$.

Maxm. *negative* (4) moment over pier (C) $= -\frac{(w + w')\,l^2}{6}$.

Max. neg. mom. at any other point,

$$\left\{\begin{array}{l} M_x = \frac{x}{6}\,(w + w')\,(2\,l - 3\,x), \text{ or} \\ M_x = \frac{w\,x}{2}\,(l - x) - \frac{x\,l}{12}\,(2\,w + w'), \end{array}\right\} \text{the greater to be taken.}$$

Any positive value of the last two, and any negative value of the first of these four equations, must of course be disregarded.

$$M_A = M_B = 0.$$

By making $M_x = 0$ in the first and last equations, and then finding the value of x, the limits of deviation of the points of contrary flexure (**23**) may be obtained.*

Fig 24.

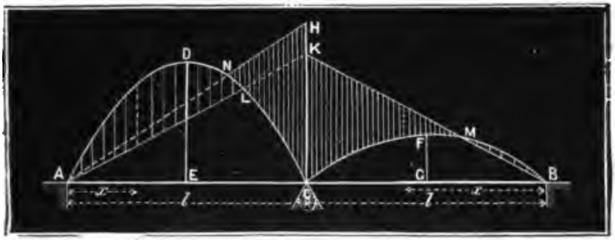

Diagram.—Let A C B be the beam (**5**, **6**). On A C draw the parabola A D C **232**), whose ordinate at centre D E $= (w + w')\,\frac{l^2}{8}$; and on C B draw the parabola C F B (**232**), whose ordinate at centre F G $= \frac{w\,l^2}{8}$. At the centre pier erect C H $= \frac{(w + w')\,l^2}{6}$, and, measuring from C, make C K $= \frac{(2\,w + w')\,l^2}{12}$. Join H to A, and K to A and B. Then the vertical distances between the parabolic arc A D L and A L will give the maximum positive (4) moments. The vertical distances between N H and the arc N L C, or those between K M and C F M, *whichever be the greater*, will give the maximum negative (4) moments. The *points of contrary flexure* (23) will approach as near the centre pier as L, and recede from it as far as M. (See foot note, p. 13.)

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* If $\left(\frac{w\,l}{2}\right)$ be not greater than $\left(\frac{(2\,w + w')\,l^2}{12}\right)$, the beam will require holding down to the abutments.

NOTE.—The various values, &c., given above, apply equally to both the spans. The diagram above is drawn to scale, on the supposition that the rolling load is $\frac{2}{3}$ the intensity of the fixed load.

33. Beam of Uniform Strength (165) for the Maximum Strains, continuous over two or more Piers, subject to a Fixed Load Uniformly Distributed, and also to a Moving Load (52). (See 113 A.)

The maximum moment over any pier will obtain, when only the two adjacent spans, and every alternate span from them, are simultaneously loaded with the greatest load, the remaining spans sustaining only the fixed load.

The maximum moment at the centre of any span will obtain when it and the alternate spans from it are fully loaded, the remaining spans sustaining only the fixed load.

Let w = the fixed or dead load per unit of length.
w' = the moving or live ,, ,,
l' = either outer span.
l = any other span.*

Then, the maximum *negative* (4) moment over any pier, B or C.,

$$M_B = M_C = -\frac{l^2}{3}\left(\frac{2\,w}{7} + \frac{w'}{2}\right).$$

Maximum *negative* (4) moment between any two piers (*i.e.*, in any inner span, l),

$$M_x = \frac{x\,(w + w')}{2}\,(l - x) - \frac{l^2}{3}\left(\frac{2\,w}{7} + \frac{w'}{2}\right),$$

(positive values of which must be disregarded).

Maximum *positive* (4) moments between any two piers (*i.e.*, in any inner span, l),

$$M_x = \frac{(w + w')}{2}\,(l - x)\,x - \frac{3\,w\,l^2}{32},$$

(negative values of which must be disregarded).

For either outer span, the maximum *negative* moments,

$$M_x = \frac{x\,(w + w')}{2}\,(l - x) - \frac{x\,l^2}{3\,l'}\left(\frac{2\,w}{7} + \frac{w'}{2}\right),$$

(positive values of which must be disregarded).

---

* If there be but three spans, a modification of the values hereafter given will be necessary, which see.

The maximum *positive* moments,

$$M_x = \frac{x\,(w + w')}{2}\,(l - x) - \frac{3\,w\,x\,l^2}{32\,l'},$$

x being measured from the abutment.

If any of the foregoing expressions for M_x be made equal to 0, the value of x obtained from them will give the positions of the points of contrary flexure (23).*

NOTE.—*If the beam be continuous for three spans only*, l, as a coefficient in the expression $\frac{l^2}{3}\left(\frac{2\,w}{7} + \frac{w'}{2}\right)$, or in $\frac{x\,l^2}{3l'}\left(\frac{2\,w}{7} + \frac{w'}{2}\right)$, must have a value given to it $= \frac{l + l'}{2}$.

Fig. 25.

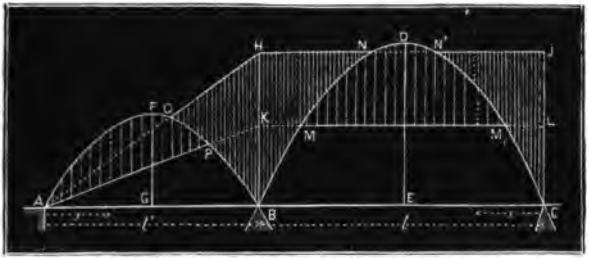


DIAGRAM.—Let A B C (5, 6) be part of the beam. On B C draw the parabola B D C (232), whose ordinate at centre (D E) $= \frac{(w + w')\,l^2}{8}$. On A B draw the parabola A F B (232), whose ordinate at centre (F G) $= \frac{(w + w')\,l'^2}{8}$. At B and C erect B H and C J† respectively, each $= \frac{l^2}{8}\left(\frac{2\,w}{7} + \frac{w'}{2}\right)$. Join A to H and H to J. Measuring from B and C, make B K and C L, each $= \frac{3\,w\,l^2}{32}$. Join A to K and K to L.

The vertical distances between A F O P and A P give the maximum positive moments for either outer span; and those between M N D N' M_1, and M M_1 give those for any inner span. The vertical distances between O H and O P B give the maximum negative moments for the outer spans; and those between H N and B M N, or between N' J and N' M_1 C, give the maximum negative moments for any inner span.

The points O and P, and M and N or M_1 and N', show the limits of deviation of the *points of contrary flexure* (23).

The diagram above is drawn to scale, on the supposition that the intensity of the rolling load is one-half that of the fixed load.

---

* If $\left(\frac{w\,l'^2}{2}\right)$ be not greater than $\left(\frac{3\,(w + w')\,l^2}{32}\right)$, the beam will require holding down to the abutments.

† N.B.—The note given above (for the value of l in the formula) must be observed here.

33 A. Note.—The moment of rupture at any point, produced by several loads acting simultaneously on a beam, is equal to the sum of the moments produced by the several loads acting separately.

SHEARING FORCES (1).

34. Abbreviations :—

Let P and P′ = the reactions on the supports due to the total load on the beam between those supports.

SH_x = the shearing force at any point x.

w = distributed load per unit of length (**61**).

W = total load concentrated at any point (**61**).

x = distance from left-hand support to the section at which S H is required.

l = length of span.

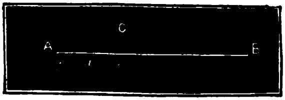
Other abbreviations will be explained where they occur.

35. In the Diagrams, the ordinates (the vertical distances from the horizontal or other lines) to the curves, &c., as shown thereon, correspond to the values of the formulæ accompanying them. If the diagram be drawn to scale in the manner as directed, the shearing forces may be obtained by *direct measurement.* See also (**6**).

36. General Rule for determining the Shearing Force at any part of a Beam and under any Distribution of Load.

Let it be required to find the shearing force at any point (C) of a beam.

Fig. 26.



Let W′ = the load between A and C.

W″ = ,, B and C.

Then, S H at C =

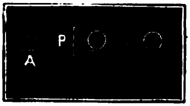
$$SH_x = P - W', \text{ or } = P' - W'';\text{—}$$

the greater of the two values to be taken.

At the supports, W′ or W″ = 0; so that the shearing forces there are always equal to P or P′. The above values hold good for semi-beams.

37. Semi-beam Fixed at One End, free at the Other, and Loaded in any Manner (8).

Fig. 27.



The shearing force at any point P is equal to all the load between that point and the unsupported extremity.

38. Semi-beam Fixed at One End only, and uniformly Loaded its entire Length (9).

$$S\,H_x = w\,(l - x). \qquad S\,H_A = w\,l.$$

Fig. 28.

Diagram.—Let A B be the beam (35). At A erect A C = $w\,l$. Join C to B. Then the vertical distances between A B and C B will give the shearing forces.

39. Beam Fixed at One End only, Loaded uniformly its entire Length, and also with a Concentrated Weight at its free Extremity (11).

$$S\,H_x = W + w\,(l-x). \qquad S\,H_A = W + w\,l.$$

Fig. 29.

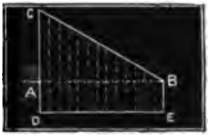


Diagram. – Let A B be the beam (35). Make A C = $w\,l$. Join C to B (as in 38). Make A D and B E = W. Join D to E. Then the vertical distances between C B and D E will give the shearing forces.

40. Beam Supported at Both Ends, and Loaded at the Centre (12).

At any point in the beam,

$$S\,H = \frac{W}{2}.$$

Diagram.—In Fig. 30, the shearing forces (35) for this case are given by the vertical distances between C D and A B, A C and B D being each $= \frac{W}{2}$ . . . (43.)

41. Beam Supported at Both Ends, and Loaded with a Concentrated Weight at any Point (13).

Let a and b be the distances of W from the supports A and B respectively.

$$S\,H_A = P = W\frac{b}{l} = S\,H_x, \text{ constant between A and W.}$$

$$S\,H_B = P' = W\frac{a}{l} = S\,H_x, \text{ constant between B and W.}$$

Diagram.—In Fig. 30, the shearing forces (35) for this case are given by the vertical distances between A B and R P Q S, W being supposed at Q.

$$A\,R = W\frac{b}{l}. \qquad B\,S = W\frac{a}{l}. \;.\;.\;. \; (43.)$$

42. Beam Supported at Both Ends, with a Concentrated Load moving in either Direction (17).

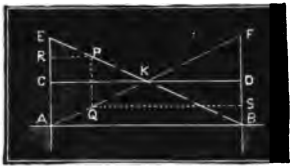
$$\left.\begin{aligned} S\,H_x &= W\,\frac{x}{l} \\ \text{Or,} \qquad &= W\,\frac{(l-x)}{l} \end{aligned}\right\} \text{The greater of these two values to be taken.}$$

$$S\,H_A = S\,H_B = W.$$

Diagram.—In Fig. 30, the verticals A E and B F at either extremity of the beam are made equal to W; E joined to B, and A to F. The vertical distances between E K F and A B give the shearing forces.

43. Diagram of the Shearing Forces in a whole Beam with a Concentrated Load.

Fig. 30.



A B is the beam. A E and B F are equal to W.

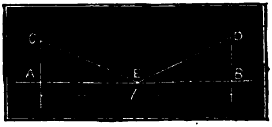
When W is in the centre of the beam (K), the shearing force for the whole length of the beam equals $\frac{W}{2}$ = the vertical distance between C D and A B.

When W is at any other point (Q), the shearing forces in the two segments, R P, Q S, are inversely as the lengths of the segments. The vertical distances between A B and R P Q S, give those shearing forces.

When W rolls from end to end of the beam, the shearing forces are as the vertical distances between A B and E K F. The points P and Q will always be points in either E B or A F.

44. Beam Supported at Both Ends, and Loaded uniformly its entire Length (19).

Fig. 31.



$$S\,H_A = S\,H_B = P = P' = \frac{w\,l}{2}.$$

$$S\,H_x = w\left(\frac{l}{2} - x\right).$$

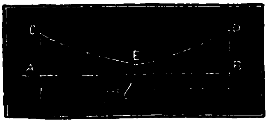
The sign of the result to be disregarded.

At mid-span, $S\,H = 0$.

Diagram.—Let A B be the beam (35). At A and B erect A C and B D, each equal to $\frac{w\,l}{2}$. Join C and D to the mid-span, E. Then the vertical distances between A B and C E D will give the shearing forces.

45. Beam Supported at Both Ends, subject to a Distributed Load advancing from either Support[*] (20).

Fig. 32.



The greatest shearing force will be developed at the point of junction of the loaded and unloaded segments; in which case let x also represent the length of the load. Then,

$$\left.\begin{aligned} S\,H_x &= \frac{w\,(l-x)^2}{2\,l} \\ \text{Or,}\qquad &= \frac{w\,x^2}{2\,l} \end{aligned}\right\}\ \text{The greater of the two values to be taken.}$$

$$S\,H_A = S\,H_B = \frac{w\,l}{2}. \qquad \text{At mid-span, } S\,H = \frac{w\,l}{8}.$$

---

[*] As in the case of a railway train crossing a bridge.

Diagram.—Let A B be the beam (35). At A and B erect A C and B D respectively, each equal to $\frac{w\,l}{2}$. Draw the semi-parabola, C E B (232), whose vertex is at B, and ordinate B D = $\frac{w\,l}{2}$. Draw the semi-parabola A E D corresponding exactly to C E B. Then, as the load advances from either pier (say A), the shearing force developed at the junction of the loaded with the unloaded apex will be represented by the vertical distances between A B and A E D. When the load covers the whole span, the shearing forces will be as in (44).

As the load is liable to advance from the pier B as well as from A, the maximum shearing forces for all positions of the load will be given by the vertical distances from A B to C E D.

46. General Formulæ for Determining the Reaction of the Supports, and the Shearing Forces in the case of Continuous Beams and Beams whose Extremities are Fixed, or subject to the Action of known Moments of Rupture (27).

Let A B be a beam, subject to the action of the moments M_A, M_B, and let the beam be uniformly loaded.

The notations as before.

$$P = \frac{w\,l}{2} + \frac{M_A - M_B}{l}.$$

$$P' = \frac{w\,l}{2} + \frac{M_B - M_A}{l}.$$

Fig. 33.

The shearing force at any point distant x from either pier, is found by subtracting $(w\,x)$ from the re-action of that pier produced by the load between A and B.

The values of P, P′ are the pressures on the piers produced by the load between A and B only. Should there be a load on the beam continued beyond these points, the pressures similarly found must of course be added to those above for the total pressures on the piers (162).

47. Beam Fixed at Both Extremities, and Loaded uniformly (25, 26).

$$S\,H_A = S\,H_B = \frac{w\,l}{2}. \qquad S\,H_x = w\left(\frac{l}{2} - x\right).$$

Exactly the same as for (44).

Diagram.—The same as in (44), which see.

48. Beam of Uniform Strength (165), supported at One End, and Fixed horizontally at the other, uniformly Loaded its entire Length (29).

$$\text{At the unfixed end, } S\,H_A = \frac{w\,l}{3}.$$

$$\text{At the fixed end, } S\,H_B = \frac{2\,w\,l}{3}.$$

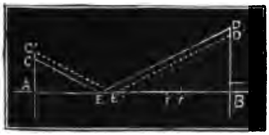
$S\,H_x = w\left(\frac{l}{3} - x\right)$, x being measured from the unfixed end. The sign of the result to be disregarded.

Diagram.—Let A B (fig. 34) be the beam (35). At the unfixed end (A) erect $A\,C = \frac{w\,l}{3}$, and at the fixed end (B) erect $B\,D = 2\,(A\,C)$. Join C and D to E, distant $\frac{1}{3}\,l$ from A, or half-way between A and the point of contrary flexure, f (23). The vertical distances between A B and C E D will give the shearing forces.

The length A f *is identical with* A f (fig. 22), *and the length* f B *with* f B (fig. 22).

49. Beam of Uniform and Equal Section, supported at One End and Fixed horizontally at the Other, uniformly Loaded its entire Length (28).

Fig. 34.



At the unfixed end, $S\,H_A = \frac{3\,w\,l}{8}$.

At the fixed end, $S\,H_B = \frac{5\,w\,l}{8}$.

$S\,H_x = w\left(\frac{3\,l}{8} - x\right)$, x being measured from the unfixed end; and the sign of the result being disregarded.

Diagram.—Let A B be the beam (35). At the unfixed end (A) erect $A\,C' = \frac{3\,w\,l}{8}$, and at the fixed end (B) erect $B\,D' = \frac{5\,w\,l}{8}$. Join C′ and D′ to E′, distant $\frac{3}{8}\,l$ from A, or mid-way between A and the point of contrary flexure, f' (23). Then the vertical distances between A B and C′ E′ D will give the shearing forces.

The length A f' *is identical with* A f (fig. 21), *and* f' B *with* f B (fig. 21).

50. Continuous Beams with Fixed uniformly Distributed Loads (30).

If the spans be rightly proportioned (see foot-note, (*) p. 12), case (48) or (49) will apply to the outer spans, and case (47) to the remaining spans.

51. Beam of Uniform Strength (165) for the maximum strains, continuous for Two equal Spans, subject to a Fixed Load uniformly Distributed, and also to a Moving Load (32). (See 113A.)

Fig. 35.

The maximum shearing force *at either abutment* will obtain when its span only sustains the moving load.

The maximum shearing force *at the centre pier* will obtain when both spans are fully loaded.

The total maximum pressure on the centre pier, when both spans are fully loaded, will be twice the above maximum shearing force at the pier.

Let w' = the moving load per unit of length, the other notations as before. The following are the maximum values[*] :—

At either abutment, $\text{S H}_A = \frac{l}{12}(4\,w + 5\,w')$, or

At the centre pier, $\text{S H}_C = \frac{2\,l}{3}(w + w')$, or

At any other point (x),

$$\text{S H}_x = \frac{l}{12}(4\,w + 5\,w') - x\,(w + w'),$$

Or, $$= \left(\frac{l}{3} - x\right)(w + w'),$$

Or, $$= \frac{l}{12}(4\,w - w') - w\,x.\dagger$$

The greatest value to be taken;† x to be measured from the abutment, and the sign of the result to be disregarded.

As the load may be supposed to advance from either abutment right across the beam, *an addition must be made* to the above values for S H_x ‡ being equal to $\frac{w'\,l}{8}$ when x is about equal to $\frac{1}{3}\,l$, and gradually diminishing as x gets more or less than $\frac{l}{3}$. (See the dotted curve in the diagram.)

DIAGRAM.—Let A C (fig. 35) be one span of the beam (35). At the abutment A erect $\text{A D} = \frac{l}{12}(4\,w + 5\,w')$. Make $\text{A E} = \frac{l}{3}(w + w)$. At C erect C F = twice A E. Join E and F to M, distant $\frac{1}{3}\,l$ from A. Through D draw D N parallel to E M. Sketch in a curve similar to that (dotted) in the figure, making an additional depth to the ordinates, at the point of minimum shearing force, of $\text{W}'\,\frac{l}{8}$. ‡ Then the vertical distances between A C and D S T F may be considered to give the maximum shearing forces for either span.

52. Beam of Uniform Strength (165) for the Maximum Strains, Continuous over two or more Piers, subject to a Fixed Load uniformly Distributed, and also to a Moving Load (33). (See 113A.)

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[*] Approximate values corresponding with the moments of rupture **32**. (See **31**.)

† The last of the three values is the shearing force in one span, when the other only is fully loaded, the addition however made for the load being a moving one will entirely cover it.

‡ An exact expression for the value of the shearing forces developed by a load gradually advancing across a continuous beam of (about) uniform strength, would, even if it could be obtained, be most complicated. The process here suggested, however, though necessarily only approximate, may be regarded as practically safe.

The maximum shearing force *at any pier* (B or C) will obtain simultaneously with the maximum moment of rupture over that pier (33).

For any inner span,

$$S\,H_B = S\,H_C - l\left(\frac{w}{2} + \frac{2\,w'}{3}\right).$$

$$S\,H_x = l\left(\frac{w}{2} + \frac{2\,w'}{3}\right) - x\,(w + w'),$$

x being measured from the nearer pier.

For either outer span—

At either abutment,

$$S\,H_A = (w + w')\frac{l'}{2} - \frac{3\,w\,l^2}{32\,l'}.$$

At the pier,

$$S\,H_B = \tfrac{2}{3}\,(w + w').$$

At any other point,

$$S\,H_x = \left(\frac{l'}{2} - x\right)(w + w') - \frac{3\,w\,l^2}{32\,l'}.$$

Or,

$$= (w + w')\left(\frac{l'}{3} - x\right).$$

The greater of these values to be taken, and x to be measured from the abutment.

At the middle of the inner spans, and at $\frac{1}{3}\,l'$ from the abutments in the outer spans, the values obtained by the above formulæ must be increased by $\frac{w'\,l}{8}$ and $\frac{w'\,l'}{8}$ respectively; and this addition, gradually diminished, must be made for some distance *on either side* of those points (see foot note (‡), p. 21) as shown in the diagram below.

If the beam be continuous for three spans only, the above formulæ must be modified as directed on the next page.

Fig. 36.

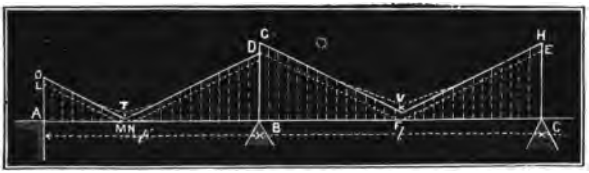

DIAGRAM.—Let A B C be part of the beam (35). For any inner span (as l).—At B and C erect B G and C H, each $= \left(\frac{w}{2} + \frac{2\,w'\,l}{3}\right)$. Make B D and C E each $= \frac{l}{2}(w + w')$. Join D and E to the mid-span, and draw G K and K H parallel to D F and F E respectively. For either outer span (as l'):—At B erect a perpendicular $= \frac{2\,l'}{3}\,(w + w')$, which, if $l' = \frac{3\,l}{4}$, will coincide with B D. At A erect A L = ½ D B. Join D and L to M, distant ½ l' from A. Make A O $= \frac{l'}{2}(w + w') - \frac{3\,w\,l^2}{32\,l'}$. Draw O N parallel to L M. At T and K sketch in curves,* similar to those in the figure, giving additional depths to the ordinates there of $\frac{w'\,l'}{8}$ and $\frac{w'\,l}{8}$ respectively. Then the vertical distances between O T D and A B may be considered to give the maximum shearing forces for either outer span, and those between G V H and B C for the remaining spans.

If the beam be continuous for three spans only, the value given in the formulæ for $S\,H_B$ and $S\,H_C$, and in the diagram for B G and C H, must be altered to $\frac{(13\,w + 16\,w')\,l}{32} + \frac{L^2}{3\,l}\left(\frac{2\,w}{7} + \frac{w'}{2}\right)$; and further, the value given to $S\,H_x$ for the inner span must be altered to $= S\,H_B - x\,(w + w')$, in which latter expression $S\,H_B$ must have the value just assigned to it.

$$L = \frac{l + l'}{2}.$$

---

* See foot note (‡), p. 21.

FLANGED GIRDERS, ARCHES, AND SUSPENSION BRIDGES.

53. In the **Design of a bridge**, girder, or other similar structure, *certain parts* of which are supposed to resist *certain strains*,* the various processes are followed out in a progressive order. When the data do not extend beyond the amount and nature of the load, and the width, &c., of the obstacle to be crossed, the processes will be as follows :—

I. *Determination of the kind of bridge* or girder (54).
II. *Determination of the general cross section, and major proportions of the structure* (55—58). These enable—
III. *An outline sketch to be drawn* (59).
IV. *Approximate estimation of the weight of the structure* (60, 61).
V. *Calculation of the strains on the various parts* (62—162) (which strains must be figured on to the outline sketch), at as many points as will be found necessary for the accurate—
VI. *Determination of the amount of material to resist the strains* on the various parts (163—173).
VII. *Distribution of the material in the various parts into a form of cross section best suited to resist the kinds of strain brought upon them* (174—190), having regard to—

  a. facility of construction.
  b. adaptation of one part to another in contact with it.

SECTION I.

DETERMINATION OF THE NATURE, PRINCIPAL DIMENSIONS, ETC., OF THE BRIDGE OR GIRDER.

(Embodying processes, I, II, III, above.)

54. The first operation (I, 53), that of determining the kind of bridge or girder, must be left entirely to the discretion of the designer; at least no rules can be laid down for his guidance: the several conditions which would influence the decision, such as nature of site, obstacles to be crossed, facilities of construction, &c., being infinitely diversified.

55. **The general cross section** of a bridge must also be left to the experience and discretion of the designer, as no definite rules can be given

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* The system of regarding particular members or parts of a girder as resisting the particular strains for which they are adapted, and these alone, is perhaps the most generally received. (See 62 and 191.)

upon the questions of the relative advantage and economy of the many systems which have been adopted and suggested.

GENERAL PROPORTIONS.

56. The central depth of straight independent girders may be made from $\frac{1}{10}$ to $\frac{1}{15}$ of the span. The greatest economy of material is perhaps obtained at $\frac{1}{12}$.

For continuous girders, or girders fixed at the ends, the depth may vary from $\frac{1}{15}$ to $\frac{1}{20}$ of the span.

57. Where convenient it is in most cases advisable that girders with fixed loads and with oblique or curved flanges or booms, should have their depths varying as the moments of rupture (**1**), that is, as the ordinates in the diagrams (**5**) given for several cases (**7—33**) of loading, &c. (**76**).

It follows from the above, that bow-string girders, arches, suspension chains with stationary uniformly distributed loads, should (the latter *will* nearly) have the form of a *parabola* (**232**).

58. Diagonal bracing will, as a rule, be most economically disposed at an angle of 45° with a vertical.

59. The design having, as supposed, advanced thus far, it is advisable that a skeleton elevation of the bridge or girder be drawn to a moderately large scale, that the strains on the various parts about to be calculated may be figured thereon.

SECTION II.

CALCULATION OF THE STRAINS ON THE VARIOUS PARTS.

(Embodying Processes IV, V, p. 24.)

60. Approximate Estimation of the Weight of the Structure.

1st. If no other source of information be at hand* (**60A**), assume a probable weight from the data of experience.

2nd. Calculate a few sections by the formulæ, &c., given hereafter, on the supposition that the bridge or girder is loaded with the just-assumed weight uniformly distributed, and the maximum extraneous load that is to be brought upon it.

3rd. Make a second approximation of the weight from the few sections just calculated; allow a percentage for contingencies (which may vary from 5 to 25), and if the total be at all near the first estimate, it will generally be sufficiently correct to stand for the weight of the structure in the final calculation of the strains, &c.

---

* See B. Baker's diagrams and tables giving weights of girders up to 200 feet span.

60A. Approximate Weights of Wrought-iron Girder Railway Bridges.

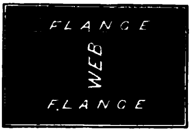
Single Line of Way.

For	30	feet spans	5	cwt. per	foot run.
,,	60	,,	6	,,	,,
,,	100	,,	9	,,	,,
,,	150	,,	12	,,	,,
,,	200	,,	15	,,	,,

61. *The weight of the beam, girder, &c., must always be introduced into the calculation for the strains.* It may generally be considered as a uniformly distributed (of course stationary) load. Where the load is stationary, and also uniformly distributed, (viz., cases **9**, **19**, **25—30**, and **38**, **44**, **47—50**), the unit of weight of the girder may be combined with that of the load proper. But in nearly all other cases it will be necessary to regard them separately.

FLANGED GIRDERS, WITH THIN CONTINUOUS WEBS.

Fig. 36A.



62. Distinct Functions of the Flanges and the Web. In pages 25 to 29 inclusive, the web is considered to take no part in resisting the *horizontal* strains (**1**), the whole of which will be provided for in the flanges.

Though this is not theoretically correct, the error is practically so small as to be disregarded with safety.

Neither are *horizontal* flanges considered to take part in resisting the shearing forces (**1**), the whole of which will be provided for in web (**64**, **80**, **191**).

Strains in Flanges Generally.

63. Flanges or parts of flanges, perpendicular to the action of the load on a girder, have to resist only the bending effect of the load, which depends on the moment of rupture (**1**).

64. When, however, a flange or part of a flange is not as just supposed, it suffers an additional strain, which is part or all (as the case may be) of the shearing force (**1**), the amount of additional strain depending on the inclination of the flange (**75**).

Thus in ordinary practical cases, where the action of the load is vertical, the strain on the flange increases the more it wanders from the horizontal position. The extra strain it suffers, however, modifies proportionally the strain on the web (**80**, **81**).

65. Nature of the strains. At any vertical section of a girder, the strains in the two flanges are of different kinds (**4**) :—

1. When the action of the load tends* to make the beam *convex* on its *lower* surface, then—

the upper flange is in compression (**168**) ; and
the lower flange is in tension (**167**).

2. When the action of the load tends* to make the beam *convex* on its *upper* surface, then—

the upper flange is in tension, and
the lower flange is in compression.

Strains in the Web Generally.

66. The strains borne by the web are the shearing forces (**1, 34**), due to the transmission of the vertical pressure of the load to the points of support. Their amount (and sometimes manner of action) are greatly modified by the longitudinal form of the adjoining flanges (**64, 80**).

GIRDERS WITH PARALLEL STRAIGHT FLANGES.

Flanges.

67. To find the Amount of Strain on either Flange at any vertical section (**68**),—**Divide the Moment of Rupture,** as found from the formulæ or diagrams, pp. 2 to 15, **by the depth of the girder,** *i. e.* by the distance between the centres of gravity of the sections of the two flanges.

68. At any vertical section, the strains on the two flanges are equal in amount, but opposite in their nature (**65**).

69. The strains in the flanges will vary throughout as the moments of rupture (**1, 2**), and therefore as the ordinates in the diagrams (**5**).

70. By Diagram.—If the ordinate, in the diagram for the case (**5**), at any point of the girder, be made to represent on a scale of parts the strain on the flanges at that point, *the strain* at any other point *may be measured off from the diagram.*

71. The strains are either direct tension (**167**), or direct compression (**168**).

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* The action of the load on a girder supported at both ends, and having its lower surface concave (see Plate I.) will lessen the concavity, and so *tend* to cause convexity.

Web.

72. The Strain upon any Vertical Section of the Web is equal to the "Shearing Force" (1, 34) developed at that section. For the value see (**34—52**).

73. If the girder have more webs than one,* the strains as found by (**72**) must be divided by the number of webs for the strain in each.

74. The nature of the strain in the web is a shearing (proper) (**171**).

GIRDERS WITH CURVED OR OBLIQUE FLANGES.

Flanges.

75. To find the Amount of Strain on either Flange at any point, —*Divide the moment of rupture* (formulæ and diagrams, pp. 2 to 15) *by the depth of the girder,* the vertical distance between the centres of gravity of the sections of the two flanges, *and multiply the quotient by the secant* (**126**) *of the angle which the flange, or a tangent to it at the point, makes with a horizontal.*†

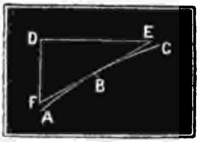
76. *If the depth of the girder vary throughout as the moments of rupture* (**1, 57**), *i. e.*, as the ordinates shown in the diagrams for those moments (**5**), then

(*a*), *the strains in the flanges will vary as the secants* (see **77, 126**) *of the angles of inclination to the horizon.* So that

(*b*), *if the strain at a horizontal part*‡ *be known, the strain at any other part may be found by multiplying the former by the secant* (see **77, 126**) *of the angle of inclination of the latter.*

77. Note. For the operation of multiplying by the secant of an angle there may be substituted a geometrical process. Let A B C be part of a curved flange. Let the value of $\left(\frac{\text{Moment of Rupture at B}}{\text{Depth of Girder at B}}\right)$ be represented to scale by the *horizontal* line DE, measuring it from some point (E) on the tangent (FE) to ABC at B. If DF be drawn vertical, *i. e.*, perpendicular to DE, then F E will $= \frac{\text{Moment of Rupture at B}}{\text{Depth of Girder at B}} \times$ secant DEF. (DEF being the angle of inclination of the flange at B.)

Fig. 37.



78. The strains in the flanges are either direct compression (**168**), or direct tension (**167**).

Web.

79. In girders with oblique or curved flanges, the strains in the web are affected by the inclinations of the flanges.

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* As in "box" or "tubular" girders.

† This is but a close approximation to the truth. See also foot-note (†), p. 61.

‡ Which will generally be at the centre of whole girders.

80.* **For the Strain on the Web at any Vertical Section,**—Find the shearing force (**1**) developed at that section (**34**), and modify it as directed below.

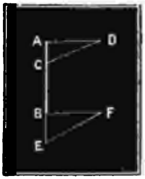
If at the vertical section the flange in compression (**65**, **78**) be inclined down $\left\{\begin{matrix}\text{to}\\ \text{from}\end{matrix}\right\}$, or the flange in tension be inclined down $\left\{\begin{matrix}\text{from}\\ \text{to}\end{matrix}\right\}$, the nearest point of support, $\left\{\begin{matrix}\text{subtract}\\ \text{add}\end{matrix}\right\}$ the vertical component of the strain in that flange $\left\{\begin{matrix}\text{from}\\ \text{to}\end{matrix}\right\}$ the shearing force. The sign of the result need not here be regarded.

If S = strain in flange,

θ = angle which the flange or a tangent to it makes with a horizontal line, as D E F (fig. 37); then,

$S \times \sin \theta$ = the "vertical component referred to above."

Fig. 38.



81. By Construction.— Draw a vertical A B = shearing force. If the flange in *tension* (**65**, **78**) be inclined down *from*, or the flange in *compression* be inclined down *to* the nearest point of support (we will suppose the latter case),—Draw C D representing both in direction and amount the strain in that flange, so that a horizontal drawn through D shall cut A. Then C A will be the vertical component of the strain in C D.

Again, if the flange in *compression* be inclined down *from*, or the flange in *tension* be inclined down *to* the nearest point of support (we will suppose the latter case),—Let it be represented by E F. Obtain its vertical component E B, which should be added to A B, in the same way that A C was subtracted from it. Then C E will be the total resulting strain on the web. Should the amount to be subtracted exceed the sum of the original and that added to it, the difference must still be taken.

81A. The strain taken must be that obtaining when the shearing force being considered is developed. For instance, with a load gradually advancing across a beam supported at both ends, the maximum shearing force will be developed at the centre, when the load covers only half the span (**45**), at which time the moment of rupture, from which the flange strain is found, will be that given by (**20**), and not by (**19**), which latter would be used when making the calculation for the flanges.

82. If the girder have more webs than one, the strain as found by (**80** or **81**) must be divided by the number of webs for the strain in each.

83. Note. It will obviously follow, from the above rules, that in girders with curved or oblique flanges, the maximum strain in the web does not necessarily occur when the maximum shearing strain is produced.

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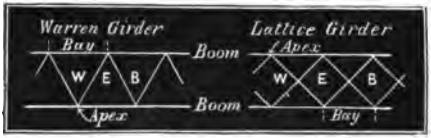
* The rules, &c., given here are not advanced as mathematically accurate, as there is much connected with the effect of curvature in the flanges on the strains in the web that has not been satisfactorily determined.

Girders with Webs of Open Bracing.

Fig. 38 A.



GENERAL RULES FOR OPEN-WEBBED GIRDERS.

Booms.

84. At any vertical section of a girder the strains in the two booms are opposite in kind. (See **65**, which also applies here.)

85. When the girder is loaded at the joints (**87**), the strain in any bay of either boom is constant throughout its length, and acts only in the direction of its length.

86. A bay cannot be in direct compression and tension simultaneously (**87**).

87. *Whenever the load or part of the load upon a girder is situated between the two extremities of any bay,* that bay must be considered as a loaded beam, and the strains in it calculated and provided for accordingly.

Web.

88. The strain on any brace is constant throughout its length, and acts only in the direction of its length.

89. A brace cannot suffer compression and tension simultaneously.

90. If two or more strains, not all of the same kind, be acting upon a brace at the same time, the total actual strain in the brace will be equal to the algebraical sum of those strains.*

91. The Nature of the Strains in the diagonal braces of girders symmetrically loaded will be—

(*a*) in all bars inclined down *to* the nearest support—*compression* (**168**).
(*b*) in all bars inclined down *from* the nearest support—*tension* (**167**).

92. If the girder be not symmetrically loaded, some of the bars will be subject to strains of a nature contrary to that stated in (**91**).

93. Counter Strains.—With a moving load, some of the bars will be subject to strains both of compression and tension, depending on the position of the load and the proportion it bears to the weight of the girder. The strain not according with (**91**) is known as the "*counter strain.*"

Note. (**92**) and (**93**) do not apply to semi-beams or cantilevers.

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* Equal to the difference between the sum of the tensile and the sum of the compressive strains.

94. If the load be brought only upon one boom,* any two bars forming an apex on the unloaded boom may be termed a "*pair*;"† if these make equal angles with the boom they are equally strained.

95. If both booms be loaded, then—

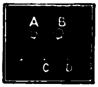
(*a*) in a warren (zig-zag or single triangle) girder there will be no "pairs."†

(*b*) in a straight lattice‡ girder, if the load be equally divided between the two booms, bars intersecting at the mid-depth of the girder, and making equal angles with the booms, may be termed a "pair," and are equally strained.

CALCULATION BY MEANS OF THE MOMENTS OF RUPTURE (pp. 2 to 15) *AND THE SHEARING FORCES* (pp. 16 to 23).

96. Loads regularly concentrated at the apices of the diagonals, may be considered as uniformly distributed loads (*w*, in 2 and 34) when the moments of rupture (1) and shearing forces are being determined, so long as the concentrated load at any apex is equal to half the sum of the supposed uniformly distributed load on the two adjoining bays. Thus the weight at A (Fig. 39) must be = $\frac{3}{4}$ that at B, before the load can be considered as uniformly distributed. If the uniformly distributed load were on the bottom boom in the fig., the weights concentrated at C and D would be equal.

Fig. 39.



GIRDERS WITH PARALLEL STRAIGHT BOOMS.

97. Note. As the depth of the girder (a constant divisor in calculating the strains from moments) is uniform, the diagrams (4, 5) may be considered to give strains instead of moments, if the vertical scale be multiplied by the number of units of length in the depth of the girder.

Thus, suppose the diagram had been drawn to a vertical scale of six tons to the inch, and that the depth of the girder was *two* feet (the foot being the lineal unit used in the case), then the ordinates in the diagram may be considered to give actual strains instead of moments if they be read off on a scale of three tons to the inch; three tons to the inch being a scale *twice* as large as six tons to the inch.

98. Note. **The Depth of the Girder** *is the distance between the centres of gravity of the cross sections of the booms,* and must always be expressed in the same units of measurement as the length of span (2, 34).

99. Warren Girder (single triangle), **Isosceles Bracing, Loaded on One Boom Only**—for any case given under Moments of Rupture, pp. 2 to 15.

Booms.§—For the strain in any bay (84—87) of the *unloaded* boom,—Divide the moment of rupture (M_x in the formulæ, or the ordinate in the

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[*] Verticals from loaded bays to opposite apices may be considered to distribute the load between both the booms.

[†] When two bars are said to form a pair, it is meant that the *same amount of the vertical pressure of the load is transmitted through them both.*

[‡] Any girder whose web consists of more than one system of triangulation is considered a "lattice."

[§] See also 97.

diagram, for the case **7** to **33**) at the opposite apex, by the depth (**98**) of the girder. For any bay (**84—87**) of the *loaded* boom, take the arithmetical mean (half the sum) of the strains in the two opposite bays of the unloaded boom.

Web.—For the strain on any pair (**94**) of diagonals forming an apex on the unloaded boom,—Multiply the shearing force ($S\,H_x$ in the formulæ, or the ordinate in the diagram, for the case **34** to **52**), developed at the apex, by the secant of the angle which the brace makes with a vertical; or increase the shearing force (as above) in the proportion of the inclined length of the brace to its vertical depth. For the counter strains (**93**) from moving loads, see (**100**).

100. The **Counter Strains** (**93**) in girders not continuous will be given by the smaller value of the two shearing forces given in **42** or **45**, or by the ordinates (35) to the lines (A K, K B. fig. 30, or A E, E B, fig. 32) in the diagrams which accompany those several cases; the values thus obtained being, of course, multiplied by the sect. (**77, 126**) of the angle between the brace and a vertical, and subject to indeed all the other stipulations made (under "Web") for the strains normal.

101. In *continuous girders with moving loads* the counter strains are indefinite, but may be supposed to act equal in intensity to the values allowed for the regular strains, and for some distance on either side of the point of minimum shearing force * (**51, 52**).

102. Warren Girder, isosceles or scalene Bracing, with the Load brought Equally upon Both Booms†—for any case given under moments of rupture, pp. 2 to 15.

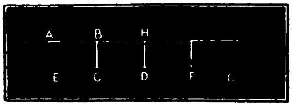
Booms.‡—For the strain in any bay (**84—87**) of *either* boom,—Divide the moment of rupture (M_x in the formulæ, or the ordinate in the diagram, for the case **7** to **33**) at the opposite apex, by the depth (**98**) of the girder.

Web.—For the strain in any brace, multiply the shearing force ($S\,H_x$ in the formulæ, or the ordinate in the diagram, for the case **34** to **52**), developed at the mid-length of the brace, by the secant (**126**) of the angle which the brace makes with a vertical; or increase the shearing force (as above) in the proportion of the inclined length of the brace to its vertical depth.

For the counter strains (**93**) from moving loads, see (**100, 101**).

103. Girder with One System of Vertical Struts and Inclined Ties, loaded either on the top or bottom, or both—for any case given under moments of rupture, pp. 2 to 15.

Fig. 40.



Booms.‡—For the strain in any bay (C D for instance) of *either* boom (**84—87**),—Divide the moment of rupture (M_x in the formulæ, or the ordinate in the diagram, for the case **7** to

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* Actually there will be counter strains for a greater length of the several spans than there will be necessity practically to provide for them in, as their amount will always be exceedingly small near the piers compared with the other strains.

† This is accomplished when there are verticals at the unloaded apices.

‡ See also **97**.

33) at the opposite apex B, by the depth (**98**) of the girder. The strain in any bay of the upper boom in a whole girder (A B, fig. 40, for instance,) will be equal to the strain in the bay (C D) of the lower boom on the mid-span side of the opposite apex; for the moment of rupture will be taken at the same point (horizontally considered) for both the bays. The bays C E and F G are unnecessary for an independent whole girder.

Web.—*Struts.* If the load be only on the *upper* boom, the compression on any strut will equal the shearing force (S H_x in the formulæ, or the ordinate in the diagram, for the case **34** to **52**), developed at the *middle of the adjoining bay on the far abutment side;** if only on the *lower*, then that at the *middle of the adjoining bay on the near abutment side;* and if *equally on the upper and lower* booms, then that developed at the strut itself.

Ties. For the tension on any inclined tie,—Multiply the shearing force (**34**) developed at its mid-length by the secant of the angle it makes with a vertical, or increase that shearing force in the proportion of the length of the tie to the length of a strut.

Counter Strains (**93**—**101**). In a girder with the bracing disposed similar to that in the figure, a moving load, or a load covering less than either half of the girder would produce tension in some of the verticals and compression in some of the diagonals (to estimate which see **100**). This can be obviated by introducing other ties (as H C and H F) which will suffer the tension otherwise brought on the verticals. These ties would, however, be useless with a stationary symmetrical load.

104. Warren Girder with scalene Bracing, Loaded on one Boom—for any case given under moment of rupture, pp. 2 to 15.

Booms.†—For any bay (**84**—**87**) in the *unloaded* boom (C D, fig. 41, for instance),—Divide the moment of rupture (M_x in the formulæ, or the ordinate in the diagram, for the case **7** to **33**) at the opposite apex (G), by the depth (**98**) of the girder.

The strain on any bay of the *loaded* boom (as E G, fig. 41) will bear the same relation to the strains in the opposite bays (H C and C D) as its apex does to their apices horizontally considered (that is, as the point F does to the points E and G). Therefore—

$$\text{Strain in E G} = \text{Str. in H C} + \frac{\text{E F}}{\text{E G}}(\text{str. in C D} - \text{str. in H C}). \quad (\text{See } \mathbf{105}).$$

Web.—For the strain in a pair (**94**) of braces forming an apex on the unloaded boom,—Multiply the shearing force (S H_x in the formulæ, or the ordinate in the diagram, for the case **34** to **52**), developed at the middle of the loaded bay included between the bars, by the secants of the respective angles which they form with a vertical; or increase the shearing force (as above) in the proportion of the inclined length of the respective braces to their vertical depth.

For the counter strains (**93**) from moving loads, see **100**.

105. Method of applying the Diagrams for the Moments of Rupture to the foregoing cases.†

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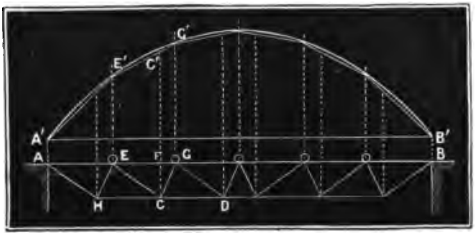
* Or the "no abutment" side in a semi-beam.

† See also **97**.

As before stated (**96**), loads which are in reality concentrated at the apices may be considered as uniformly distributed if they be such as a uniformly distributed load would bring on the apices in question.

Fig. 41.



Place the diagram of the moments of rupture (**1**) for the case of loading, support, &c. (**7** to **33**) immediately above (or under), and drawn to the same horizontal scale as the skeleton elevation (**53**, **59**) of the girder. (See fig. 41, in which the load is supposed to be uniformly distributed (**96**, **19**) along the upper boom.)

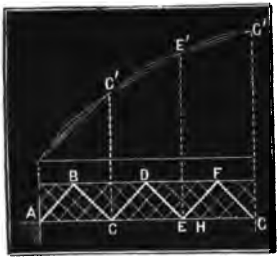
Draw verticals right through the diagram at each of the *loaded* apices, whether on one or both booms (as E E′ and G G′ fig. 41.) If the lines to or from which the ordinates in the diagram are supposed to be taken be curved, draw chords to that curve at the intersections of the verticals just drawn with that curve (as in fig. 41.) A polygon will be thus formed, and the strain on any bay of either a loaded or unloaded boom will be given (**97**) by the ordinate to that polygon taken at the opposite apex. (See also **106**.)

For the diagonals the shearing forces may be taken directly from the diagram for the case (**34** to **52**) drawn to a large scale (**35**), subject to the modifications given under "Web" in the cases above.

106. Note. If all the lines in the diagram for the moments of rupture, to or from which the ordinates are directed to be taken, be *straight*, then the strain in any bay of *either* boom, whether *loaded* or *unloaded*, may be found from the ordinate taken at the opposite apex, by dividing the value of that ordinate by the depth of the girder (unless **97** be complied with, when the latter operation must, of course, be omitted).

107. Lattice Girder *without a central bay, and if of more than two systems of triangles then with a complete system of triangulation in the half span (as* A B C D E F G), *but any number of intermediate systems* **loaded uniformly on one boom for an equal distance on either side of the mid span** (see **110**): may be applied also to cases of continuous girders (**30**—**33**).

Fig. 42.



Booms.—Place the digram (**5**, **6**, **97**) for the moments of rupture (**1**) for the case above (or below) the outline sketch of the girder (**59**) as in fig. 42.

Draw chords and tangents (as shown in fig.) at points (C′ E′ G′) corresponding with the divisions made by the "primary triangles." Two polygons will thus be formed, one by (or including*) the chords, another by (or including*) the tangents. For the strain in any bay of the *unloaded* boom take the ordinate to the *circumscribed* polygon,* at the centre of the bay; and for any bay (as E H) of the *loaded* boom take the ordinate to the *inscribed* polygon* at the centre of the bay. If the ordinates represent the moments of rupture (**4**, **5**, **97**), divide this value by the depth (**98**) of the girder.

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* When (**21**) is applied the load being centrally situated, the chords and tangents will of course extend only along the parabola C E D (fig. 15).

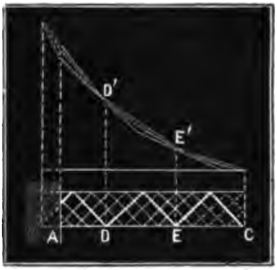
Web.—Divide the load per unit of length (**34**) by the number of systems of triangles (= the number of bays in the base of one of the primary triangles) for a new unit of weight, w_1; thus $\frac{w}{n} = w_1$ or $\frac{w'}{n} = w'_1$. Then for the strain on any pair (**95**) of lattice bars,—Multiply the shearing force ($S\,H_x$ in the formulæ, or the ordinate in the diagram—**34**, **35**,—calculated or constructed with the new unit w_1, or w'_1,) developed at their apex by the secant of the angle they make with a vertical, or increase that shearing force in the proportion of the inclined length of the bar to its vertical depth.

108. Any Lattice Semi-girder loaded on One Boom only.

Booms.—If the load be concentrated, see **110**. If distributed, place the diagram for the moments of rupture (**4**, **5**) immediately above (or below) the outline sketch of the girder (**59**).

Trace out the system of triangulation which terminates fairly at the extremity of the girder (see the thick lines in the fig.) To the curve of moments (**4**, **5**) draw chords and tangents at points (E′, D′) corresponding to the divisions made by the triangles just traced out (D, E, fig. 43). If necessary for the construction, the diagram may be continued into the abutment. Two polygons will thus be formed, when, whatever be the number of systems of triangulation,—For the strain in any bay (**84**—**87**) of the *unloaded* boom take the ordinate, at the centre of that bay, to the *circumscribed* polygon; and for any bay of the *loaded* boom, to the *inscribed* polygon. If the ordinates represent the moments of rupture (**4**, **5**, **97**), divide their value by the depth of the girder (**98**).

Fig. 43.



Web.—If the load be concentrated, see (**110**). If distributed, divide the load per unit of length (w) by the number of systems of triangles (= the number of bays in the base of a primary triangle (4 in fig. 43) for a new unit of load $\left(\frac{w}{n} = w_1\right)$. Then for the strain on any pair (**95**) of braces forming an apex on the unloaded boom,—Multiply the shearing force ($S\,H_x$ in the formulæ, or the ordinate in the diagram for the case, calculated or constructed with the *new unit* w_1—**34**, **35**), developed at their apex, by the secant of the angle which they (each) make with a vertical, or increase the shearing force (as above) in the proportion of the inclined length of the brace to its vertical depth.

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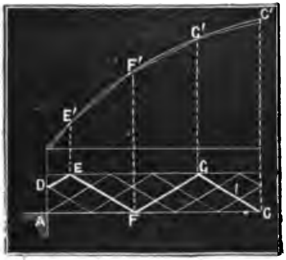
* See note, p. 34.

100. Lattice-Girder, *if whole, and of more than two systems of triangles, then not without a central apex on one or other of the booms* (see fig. 44), **loaded uniformly and equally on both the Booms for an equal Distance on either side of the Mid-span,** may be applied to continuous girders (**30**—**33**).

Booms.—Place the diagram (**4, 5**) of the moments of rupture (**1**) immediately above (or below) the outline sketch (**53, 59**) of the girder (as in the fig., where the load is supposed to extend over the whole length of the girder).

Fig. 44.



Where the line to or from which the ordinates are taken is curved, draw chords to it at points (E′ F′ G′ C′) corresponding to the several apices (on both booms) of the system of triangulation which has an apex at the mid span (E F G C in fig.) For the strain in any bay (**84**—**87**) of either boom, take the ordinate to the polygon,* thus formed, at the centre of that bay. If the ordinates represent the moments of rupture (**4, 5, 97**), divide their value by the depth of the girder.

Web.—Divide the load per unit of length (w) by the number of systems of triangulation (= the number of bays in the base of one of the primary triangles,—3 in fig. 44) for a new unit of load $\left(\frac{w}{n} = w_1\right)$. Then for the strain in any lattice bar, or pair † of bars. Multiply the shearing force (S H, in the formulæ, or the ordinate in the diagram for the case—**34, 35**), developed at its or their mid-length, by the secant of the angle it or they (each) make with a vertical, or increase the shearing force (as above) in the proportion of the inclined length of brace to the vertical depth.

109A. *If there be only Two Systems of Triangulation.* Then for the strain in any bay (**84**—**87**) of either boom. Take the arithmetical mean between the moments of rupture at the two ends of the bay (p and q), and divide it by the depth of the girder.

$$\text{Strain in any bay} = \frac{M_p + M_q}{2d}.$$

The chords in the diagram will, however, give the same result.

110. Concentrated Loads on Lattice Girders. If a lattice girder be subjected to the action of a concentrated load at any apex, as at D (fig. 42), the system of triangles upon one apex of which the load is situated (A B C D E F G, &c.) should be considered as constituting the sole web of the girder—that is, as far as the concentrated load is concerned, for there is also the distributed weight of the girder (**61**). The strains from the concentrated load might be calculated as if the girder were a warren, and as if the intermediate bracing did not exist. Indeed, to introduce other systems of triangles for a stationary concentrated load would be an error in design, unless the boom upon which the load is placed be made sufficiently rigid to distribute part of the pressure upon adjacent intermediate apices, in which case the strains would be very indefinite.

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* If it be an application of case (**21**) the lines A F and G B (fig. 15) must be considered to form part of the polygon.

† There will be no pairs (**95**), unless the number of systems of triangles be even

GIRDERS WITH CURVED OR OBLIQUE BOOMS.

111. Any Curved or Oblique whole or Semi-girder with a Single-triangular Web, loaded on One Boom.* (This will include many roof principals, bow-string girders, bent cranes, &c.)

BOOMS.—If alternate braces be vertical, then for the strain in any bay of *either* boom; if not, then for the strain in any bay of the *unloaded* boom *only*,—Divide the moment of rupture (M_x in the formulæ, or the ordinate in the diagram (**4**, **5**), for the case **7** to **33**) at the opposite apex by the length of the perpendicular let fall from that apex on to the bay. For the strains in the *loaded* bays, when all the braces are inclined, the best way is by the diagram as follows. Place the diagram of the moments of rupture, immediately above (or below) the outline sketch (**53**, **59**) of the girder. If the lines in the diagram to or from which the ordinates are directed to be taken be curved, draw chords at points corresponding to the position of the apices in the *loaded* boom. Then for the strain in any bay of the *loaded* boom, divide the value of the ordinate to the just-formed polygon, taken at the opposite apex, by the length of a perpendicular let fall from that apex on to the bay (**121**). (See **154.**)

WEB.—For any brace, first determine the shearing force (**34**), acting at the middle of the bay on the loaded boom, which forms part of the same triangle with the brace in question. Next, find the vertical components (**80**, **81**) of the strains in those bays which are opposite sides of a quadrilateral figure, whose diagonal is the brace in question. If either of these bays be part of a boom in tension, and sloped as a strut (**91**), or part of a boom in compression and sloped as a tie (**91**), *add* the vertical component of the strain in it (already found) to the shearing force first obtained. Again, if either of the two same bays be part of a boom in compression and sloped as a strut, or part of a boom in tension and sloped as a tie, *subtract* its vertical component (already found) from the result of the last operation (the addition, if any). The total resulting quantity must then be multiplied by the secant of the angle the brace makes with a vertical, or increased in the proportion of the inclined length of the brace to the vertical distance between its ends. *If the sign of the result be negative* (—), it shows that the nature of the strain on the brace is opposite to that which its position would have indicated according to the general rules (**91**). *If the brace be horizontal*, the shearing force must be disregarded, and the horizontal components of the bays substituted for their vertical components in the process detailed above. The strain in a horizontal bay can have no vertical component.

112. Any Curved or Oblique whole or Semi-girder with a single-triangular Web, loaded equally on Both Booms.

BOOMS.—For the strain in any bay of *either* boom,—Divide the moment of rupture (M_x in the formulæ, or the ordinate in the diagram (**4**, **5**) for the case **7** to **33**), taken at the opposite apex, by the length of a perpendicular let fall from that apex on to the bay. (See **154.**)

WEB.—As in the last case, excepting that the shearing force must be taken at the mid-length of a line joining the centres of the bays, which are two opposite sides of a quadrilateral whose diagonal is the brace.

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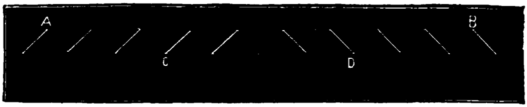
* See foot note (*), p. 31.

113. For Continuous Girders, and Girders fixed at the Ends, the foregoing methods may be used in connection with cases **24** to **33** and **46** to **52**, provided that the conditions stated in the latter and below (**113A**) are complied with.

113A. Wherever there is a negative moment of rupture at (or for safety in practice *near*) the abutments, the girder should be anchored down at its extremities. (See foot notes, pp. 13 and 15.)

114. Fixing the Points of Inflexion of Continuous Girders.—The points of inflexion (**23**) may be practically fixed at any part of a continuous girder subject to a moving load, by severing either boom at that part; if the upper boom, the parts thus severed must be prevented from coming in contact. *The structure is thus resolved into a series of independent girders, the strains upon which can then be most readily calculated.*

Fig. 45.



Example.—In the accompanying fig. (45), by severing (or really removing) the bays of the upper boom opposite C and D, A C and D B become virtually semi-girders, each having to sustain in addition to the distributed load upon its length, half the total load on C D, suspended at the extremity (**11**, **39**). C D is simply an independent girder supported at both ends.

115. *The points of inflection may be considered as fixed* in those continuous girders, and girders fixed (or tied back) at the ends, whose depths vary as (or nearly as) the moments of rupture. The strains in these also may be calculated as if the several divisions were independent girders.

If the tension members running down from the towers (as in fig. 46) be made to act simply as suspension chains, the strains on them may be obtained from **155** or **118**.

Fig. 46.

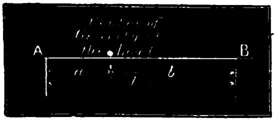
CALCULATION BY THE COMPOSITION AND RESOLUTION OF FORCES.

(APPLICABLE TO ALL CASES OF OPEN-WEBBED INDEPENDENT GIRDERS.)

116. The following principles should be applied to the calculation of the strains on the various members of an open girder (discontinuous) *by* (1*st*) *finding the reaction of the supports from any weight in the girder, and* (2*nd*) *tracing this reactionary pressure throughout the various parts.*

117. Reaction of Supports. Let a and b equal the distances from the supports A and B, respectively, of the centre of gravity of a load (W) on a beam. Then,

Fig. 47.



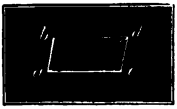
Pressure on A = Reaction of A = $W \frac{b}{l}$.

Pressure on B = Reaction of B = $W \frac{a}{l}$.

In girders, properly so called, the supports are supposed to be capable of resisting vertical pressure only. Their reaction can then only be vertical, and this must be borne in mind.*

118. Composition and Resolution of Forces. If two forces ($a b$ and $a c$) acting at a given point (a), be represented both in direction and amount by the two adjacent sides ($a b$, $a c$) of a parallelogram, an equivalent force will be represented in both direction and amount by the diagonal ($a d$) of that parallelogram.

Fig. 48.



The converse of this is also true.

119. If three forces, acting at a given point, be in equilibrium (balance), their direction and amount will correspond to the three sides of a triangle, if any two of which be given, the third may be found. This triangle is nothing more than $a b d$ or $a c d$, fig. 48.

120. If there be more than one concentrated load on a girder, two courses may be followed :

The reactions and the strains produced by each weight separately may be found, and the algebraical sum (see foot note, p. 30) taken ; or,

The reaction of the piers from the whole load may be worked towards the mid-span, the downward pressures of the several weights (when they are met with), being compounded with the other pressures during the progress. See Example (**122**).

The first should be adopted when the load is variable—for then the maximum resulting strains of either kind on any member may be found (**136**) ; the second when the load is stationary.

EXAMPLE—

Strains in a Bent Girder (*Roof Principal*).—Plate I.

121. *Calculation by the Moments of Rupture* (**1** *to* **33**) *and Shearing Forces* (**34** *to* **52**).

A F K is half of a bent girder whose clear span is 80 feet. The lower boom consists of chords to a curve of 96 feet radius; and the upper, chords to a curve of 57 feet radius. The load is considered as uniformly distributed (regarded horizontally) on the upper boom, and = 8 tons = ·1 ton (w) per foot of span. The braces are alternately vertical and inclined. (See **111**.)

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* Where the structure *abuts against* the supports, and those supports are supposed to be capable of resisting the lateral pressure, the structure is virtually an arch, and the reaction of abutments must not be considered solely as vertical. (See **146**.)

Strains in the Booms.—The strain in the upper boom is compressive, and in the lower tensile (**65**). As the braces are alternately vertical and inclined (**111**), the strains in any bay of either boom will equal the moment of rupture (**1**) for the case (**19**) taken at the opposite apex, and divided by the depth of the girder taken as a perpendicular from the apex on to the bay. (See Plate I.)

The moments of rupture (M) may be obtained either from the formula $\left(M = \frac{wx}{2}(l-x) \quad (\mathbf{2})(\mathbf{19})\right)$, or, from a properly constructed diagram (**4, 5**). The depths (as above) must be scaled from the outline sketch (**53—59**).

The diagram for the moments as given in **19** apply properly to the unloaded boom only; while for the loaded boom, an inscribed polygon is required (**111**), as shown in Plate I. But wherever the apices on one boom are vertically under (as in the present instance) or over those on the other boom, it will be evident that the only ordinates required from the polygon will be those taken to its angles, that is, to the very points where it coincides with the parabola: so that the polygon is unnecessary.

It will be seen that by drawing the lines of construction for the parabola, as dotted in on the Diagram, there is really no necessity to draw the actual curve. (See also note to **232**, III.)

$$\text{Strain in any bay} = \frac{M}{d} = \frac{w\,x(l-x)}{2\,d} = \frac{\text{Diagram Ordinate}}{d}.$$

By Formulæ. *By Diagram.*

$$\text{Strain in A B} = \frac{\cdot 1 \times 5 \times 75}{2 \times 1{\cdot}6} = \frac{18{\cdot}75}{1{\cdot}6} = 11{\cdot}7 \text{ tons.}$$

$$,, \quad \text{B C} = \frac{\cdot 1 \times 13{\cdot}75 \times 66{\cdot}25}{2 \times 3{\cdot}9} = \frac{45{\cdot}53}{3{\cdot}9} = 11{\cdot}6 \text{ tons.}$$

$$,, \quad \text{C D} = \frac{\cdot 1 \times 22{\cdot}25 \times 57{\cdot}5}{2 \times 5{\cdot}7} = \frac{64{\cdot}68}{5{\cdot}7} = 11{\cdot}3 \text{ tons.}$$

$$,, \quad \text{D E} = \frac{\cdot 1 \times 31{\cdot}25 \times 48{\cdot}75}{2 \times 6{\cdot}9} = \frac{76{\cdot}17}{6{\cdot}9} = 11{\cdot}039 \text{ tons.}$$

$$,, \quad \text{E F} = \frac{\cdot 1 \times 40 \times 40}{2 \times 7{\cdot}25} = \frac{80}{7{\cdot}25} = 11{\cdot}031 \text{ tons.}$$

$$,, \quad \text{A G} = \frac{18{\cdot}75^{*}}{1{\cdot}9^{\dagger}} = 9{\cdot}8 \text{ tons.}$$

$$,, \quad \text{G H} = \frac{18{\cdot}75^{*}}{2{\cdot}0} = 9{\cdot}3 \text{ tons.}$$

$$,, \quad \text{H I} = \frac{45{\cdot}53^{*}}{4{\cdot}5} = 10{\cdot}1 \text{ tons.}$$

$$,, \quad \text{I J} = \frac{64{\cdot}68^{*}}{6{\cdot}1} = 10{\cdot}6 \text{ tons.}$$

$$,, \quad \text{J K} = \frac{76{\cdot}17^{*}}{7{\cdot}08} = 10{\cdot}7 \text{ tons.}$$

Strains in the Web.—The values of the Shearing forces (**1, 34**) may be obtained either from the formula for the case (**44**) or from a diagram (**35**), the

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[*] The moments of rupture are the same as for the upper boom, as the apices are at the same horizontal distance from the supports.

[†] This "depth" is a perpendicular, let fall from B on to A G produced. (See Plate.)

triangle A′ B′ C′ (Plate I.) corresponding to A C E (fig. 31). The vertical components of the strains in the booms must be obtained (**111**) either by (**80**) multiplying such strains by the sines of the angles of inclination of the several bays (as the angle *i j* D for the bay D E, Plate I.), or by a geometrical construction, as show for the bays D E and I J; for the former, D *j* is the strain in the bay, and D *i* the vertical component.

$$\text{Strain in any brace} = \left\{ \text{S H} \pm (\text{vert. comp.}) \pm (\text{vert. comp.}) \right\} \sec.\,\theta.$$

Strain in F K = ·437 — ·85 = — ·413 tension.*
,, K E = (·437 + ·48 — ·85) × 1·7 = ·114 tension.*
,, E J = 1·3 + ·48 — 2·6 = — ·82 tension.
,, J D = (1·3 + 1·4 — 2·6) × 2 = ·218 tension.
,, D I = 2·2 + 1·4 — 4·3 = — ·7 tension.
,, I C = (2·2 + 2·22 — 4·3) × 3·5 = ·42 tension.
,, C H = 3·07 + 2·22 — 6·2 = — ·93 tension.
,, H B = (3·07 + 3·0 — 6·2) × 9·5 = — 1·24 tension.
,, B G = Diff. bet. vert. comp. of A G and G H.
= 3·0 — 3·7 = — ·7 tension.

122. *Calculation by the Composition and Resolution of Forces* (**116—120**).

For the data see **121**. The weights (**125**) on the several apices of the upper boom must be considered as half the load on the adjacent bays, the load being horizontally uniformly distributed. Thus at B will be (2·5 + 4·375) × ·1 tons = ·687 tons; and on each of the remaining apices on the upper boom 8·75 × ·1 = ·875 tons. The sum of all these weights on the girder will be 7·5 tons, the other ·5 ton being supported directly by the two piers (·25 ton on each, being half the load on the outer bay). The reaction (**117**) of either pier from the total load on the girder will be $\frac{7{\cdot}5}{2} = 3{\cdot}75$, and this must be worked up to the centre by the application of (**118** or **119**).

At pier A (Plate I.) draw a perpendicular (A *b*) = the reaction of A as above. Produce A B to *a*, and through *b* draw *b a* parallel to A G. A *b a* is the triangle of forces at A. Then (**119**) *b a* is the tension in A G; and A *a* the compression in A B. Through *a* draw *a c* parallel to G H; then *a c b* is the triangle of forces at G; *c a* is the tension in G H, and *c b* that in G B. At B there are acting the compression in A B, the tension in G B, and the vertical pressure of the weight (·687 tons) at B, all of which are held in equilibrium by B C and B H. Let A *d* = the sum of the tension (*c b*) in G B and the weight at B. Join *d* to *a*; and *d a* is the resultant of the three forces just named. To avoid confusion, repeat *d a* at *a d′*. Through *d′* draw *d′ e* parallel to C B; and through *a*, *a c* parallel to B H. Then *e d′* is the compression in B C; and *a e* the tension in B H. Transfer *a e* to *c f*, and join *f* to *a*; then *f a* is the resultant of the strains in B H and G H; which resultant may be resolved in the directions of H C and H I; and so on to the centre.

The inexperienced practitioner will find it more convenient and safe to work by the parallelograms (**118**) *instead of the triangles of forces*, as the nature of the resulting strains is more clearly shown by the former, and there is also less liability to confusion.

METHODS OF CALCULATION FOUNDED ON THE PARALLELOGRAM OF FORCES.

123. General Law of the Strains in the Booms of Horizontal Straight Girders. The increment of strain developed at any apex in the boom, is equal to the resultant of the horizontal components of the strains in the two diagonal bars forming the apex.

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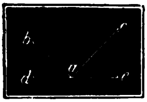
* See **111**, "Web."

Fig. 49.



Let *a b* and *a c* represent in direction and amount the strains in two bars forming an apex, one bar being in compression, the other in tension. Let the verticals *b d* and *c e* be drawn, and through *a*, the horizontal *d e*. Then *b d* and *c e* will correspond to the load on the diagonals *b a* and *a c* respectively; *d a* and *a e* will be their horizontal components; and *d e* will be the increment of strain developed at *a*.

When, however, the strains in the bars are both tension or both compression, their horizontal components will be antagonistic, and the increment will then be the difference between them instead of their sum (as *d e*).

A vertical brace has no horizontal component.

124. *The general Rules for booms and bracing* (**84** to **95**) *hold good here also.*

125. If the load be not concentrated at the apices, each apex must be considered to sustain part of the load on the two adjacent bays. If the load be uniformly distributed, each apex will sustain half the load on the two adjacent bays. For other distributions of load, the pressure or equivalent weight on each apex must be determined from **117**. The total loads thus allotted to the several apices will be called the "weights."

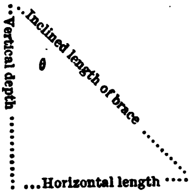
126. In this division of the work—

Let θ = Angle made by any brace with a vertical line. Then—

$$\text{Sec. } \theta = \frac{\text{Inclined length of brace}}{\text{Vertical distance between its ends}}.$$

$$\text{Tan. } \theta = \frac{\text{Horizontal distance between its ends}}{\text{Vertical distance between its ends}}.$$

$$\text{Sin. } \theta = \frac{\text{Horizontal distance between its ends}}{\text{Inclined length}}.$$



127. Straight Semi-girder loaded in any manner.

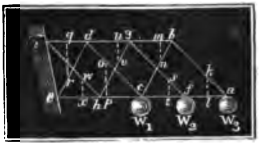
WEB.—*Every weight* (**125**) *on the girder is transmitted through its own system of triangles to the abutment. The load on any brace* is equal to the sum of all the weights upon its system, between it and the unsupported end of the girder. For the *strain* on any brace multiply the load on it (as above) by sec. θ.

BOOMS.—By (**123**),—The horizontal component of the strain in any brace = (strain in brace) × sin. θ. (**126**.)

128,—*EXAMPLE.*—Let fig. 50 represent a parallel straight girder of otherwise irregular construction loaded at three points (*a f c*). The weight of the load at *a* is conducted along the braces *a b c d e*, that at *c* along *c d e*. If *l k* represent the weight W_3, $k\,a$ will be the tension (**91**) on $a\,b$, and $l\,a$ = increment (**123**) at a. If $m\,n$ also equal W_3, $n\,b$ will equal the compression in $c\,b$, and $m\,b$ the increment at b. The load on $c\,d$ is equal to W_1 and W_3 together. If $o\,p$ = this load, $o\,c$ will equal the tension on $d\,c$, and $p\,c$ the increment at c, and so on; $s\,t$ being equal to W_2, $s\,f$ will equal the strain in $f\,g$, and $t\,f$ the increment at f.

Then (**123**) the tension (**65**) in $b\,g = m\,b + l\,a$; that in $d\,g = u\,g + t\,f +$ strain in $b\,g$; that in $i\,d = q\,d + p\,c +$ that in $d\,g$. So also the compression (**65**) in $a\,f = a\,l$; that in $f\,c = t\,f +$ that in $a\,f$; that in $c\,h = c\,p + m\,b +$ that in $c\,f +$ that in $f\,a$; and so on. If the triangles for the strains be drawn to a large scale, the strains may be obtained quickly and accurate.

Fig. 50.



129. Straight Warren Semi-girder, Isosceles Bracing, loaded at the Extremity.

Web.—Strain (Σ) in any brace = W sec. θ. . . . (**126.**)

Booms.—Strain (S) in any bay of either boom,

$$S = n\,W \tan.\ \theta, \quad . \quad . \quad . \quad (\mathbf{126,})$$

where W = the weight, and n = number of diagonals between the centre of the bay and W.

130. Straight Warren Semi-girder, Isosceles Bracing, loaded uniformly on One Boom.

The weight on the end apex = half that on the others (**96**).

Web.—Strain (Σ) in any pair of diagonals forming an apex on the unloaded boom.

$$\Sigma = w\,x \sec.\ \theta, \quad . \quad . \quad . \quad (\mathbf{126,})$$

where w = load per unit of length, and x = number of units between the apex and the unsupported end of the girder.

Booms.—The strain (S) in any bay of the loaded boom,

$$S = \left\{ m\,(m-1) + \tfrac{1}{2} \right\} w\,y \tan.\ \theta \quad . \quad . \quad . \quad (\mathbf{126.})$$

In any bay of the unloaded boom,

$$S = m^2\,w\,y \tan.\ \theta, \quad . \quad . \quad . \quad (\mathbf{126,})$$

where m = number of the bay counted from the outer end of the girder, and y = length of bay.

The following cases (viz., **131** to **138** inclusive) do not apply to continuous girders or whole girders fixed at the ends.

131. Straight Warren Girder, Isosceles Bracing, loaded at Any Point.

Web.—Strain (Σ) in any diagonal between A and W,

$$\Sigma = W \frac{b}{l} \text{ sect. } \theta.$$

In any diagonal between B and W,

$$\Sigma = W \frac{a}{l} \text{ sec. } \theta \quad . \quad . \quad . \quad (\mathbf{126.})$$

Fig. 51.

Booms.—Strain in either end bay of the longer boom = (reaction of support) × tan. θ (**117**, **126**); and if this value be considered a unit, then the strains in the bays of the longer boom will be proportional to the series 1, 3, 5, 7, &c.; and in the shorter boom to the series 2, 4, 6, 8, &c., counting from the supports. Thus the strain in the second bay of the longer boom, from pier B will equal $\left(3\ W\ \frac{a}{l}\ \tan.\ \theta\ \right)$.

Strain in the bay opposite the loaded apex $= \frac{W\,a\,b}{l\,d}$;—d being the depth (**98**) of the girder.

132. If W be in the centre of the girder, the strain (Σ) on any diagonal,

$$\Sigma = \frac{W}{2}\ \sec.\ \theta \quad . \quad . \quad . \quad (\mathbf{126}.)$$

And the strain (S) on the centre bay,

$$S = \frac{W\,l}{4\,d}.$$

133. Straight Warren Girder, Isosceles Bracing, with a concentrated Moving Load.

WEB.—Every diagonal except the two end ones will be subject to counter strains (**93**). The maximum strain normal on any pair of braces forming an apex on the unloaded boom, will occur when the load is at the inner (mid-span) end of the inclosed bay. The maximum counter strain on any pair will obtain when the weight is at the outer (near abutment) end of the enclosed bay. The values can be obtained from (**131**).

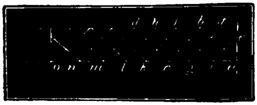
BOOMS.—The maximum strain on any bay (**84-7**) of the unloaded boom is when the weight (W) is at the opposite apex; and on the loaded boom when the weight is at the next inner apex. The values can be obtained from (**131**).

134. Any Straight Warren or Lattice Girder (isosceles of scalene bracing), **with any Load symmetrically disposed about the Centre,** either on one or both booms.

Suppose the load collected at the apices, according to (**125**).

WEB.—*The load on any bar will be equal to the sum of all the weights* (**125**) *on its system of triangles, between it and the centre of the girder.* The *strain* on the bar will equal the load on it, multiplied by the secant of the angle it makes with a vertical (**126**).

Fig. 52.



Let the accompanying fig. represent half a girder, with loads on both booms symmetrically disposed about the centre *a*. Then the load on *b a* will be half the weight (**125**) at *a*, that on *b c* half the weight at *a*, and the whole weight at *b*; that on *c d*, the weights at *c* and *b* and half the weight at *a*, and so on; *f g* will take the whole weight at *f*; *i j* the whole weight at *i*; *e f* and *i e* will not be necessary. (See foot note, p. 46.)

BOOMS.—The strains on the several bays will be best obtained from the web, by the application of (**123**), by summing the several increments (commencing at the outer ends of the girder) as follows.

The strain in *o n* is the increment at *o*. But the strain in *o n* pervades the whole length of the bottom boom, and therefore must be added to all the other increments *inwards* (towards the centre of the girder). The same may be said of the strains in the other bays; so that, strain in *n m* = (inc. at *o*) + (inc. at *n*

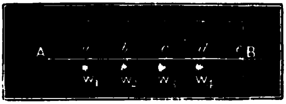
Strain in $n\,l$ = (inc. at o) + (inc. at n) + (inc. at m) = (strain in $m\,n$) + (inc. at m). Therefore, *the strain in any bay is equal to the strain in the next outer bay* (+) *plus the increment at the intermediate apex.* Note. This applies also to semi-girders.

135. Any Straight Warren or Lattice Girder (isosceles or scalene bracing), **with a uniformly distributed Moving Load.***

Web.—The simplest method of obtaining the strains and counter strains (**93**) is by tabulating the strains produced by each weight separately, using (**131**, "Web"), which will hold good for other than isosceles bracing, provided the sec. θ (**126**) is corrected for the varying angles. The co-efficient, sec. θ, should not, however, be employed until after the summary is made. See example (**136**).

Booms.—The strains in the booms are greatest when the girder is fully loaded, and may be most easily obtained from the web by (**123** and **134**).

Fig. 53.



136. *Example.*—A B, fig. 53, is a girder 50 ft. long, 10 ft. deep. The lattice bars are inclined at angle of 45° with a vertical. The moving load is equal to 1 ton per lineal foot, = 10 tons at each apex (on the lower boom) (**125**). In tabulating the strains, only those bars which incline in one direction need be considered, for the remainder are strained similarly and equally. (Thus a, fig. 53, corresponds exactly with the bar intersecting e, and so on). In the table below, + indicates compression, and — tension. The numbers are the *loads* on the bars. For the *strains*, the loads must be multiplied by sect. 45° = 1·4. The columns (maximum +) and (maximum —) are obtained by adding the + values together and the — values together respectively in each horizontal row. The column (uniform load) is the algebraical sum of the values in the horizontal rows.

Tracing the action of W_1, it is seen (from **131**) that in a, fig. 53, there is from it a load $= W\frac{b}{l} = 10 \times \frac{40}{50} = 8$ tons, evidently producing tension (—); and in c and e there is a load $= W\frac{a}{l} = 10 \times \frac{10}{50} = 2$ tons, as evidently producing compression (+). And so on for the other weights.

Bars.	W_1	W_2	W_3	W_4	Maximum +	Maximum —	Uniform Load.
a	— 8		— 4			— 12	— 12
b		— 6		— 2		— 8	— 8
c	+ 2		— 4		+ 2	— 4	— 2
d		+ 4		— 2	+ 4	— 2	+ 2
e	+ 2		+ 6		+ 8		+ 8

It will be seen from the above Table that bars c and d (and of course the bars inclined the other way which correspond to them) will suffer strains of both tension and compression.

---

* Case of a railway train passing over.

137. NOTE.—The strains in the web resulting from the weight of the girder itself, must not be calculated simultaneously and in combination with those resulting from the moving load. They should be calculated as for a stationary load (either by **134**, or by tabulating as above, to form the column "uniform load").* The values thus obtained for the several bars should then be algebraically added to each of the columns ("maximum +", "maximum —", and "uniform load"); and it will be found that the modification in the first two will be more advantageous as the weight of the girder becomes large compared with that of the moving load; for the counter strains (**93**) will be less, and the amount of counter bracing necessary will be diminished also.

138. Any Straight Open-braced Girder, with a load unsymmetrically disposed about the Centre.

WEB.—*a*. By tabulating the strains produced by the several weights, and taking their algebraical sum;

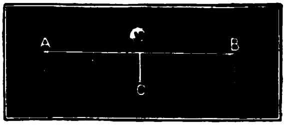
Or, *b*. By (1st) abstracting the unsymmetrical parts of the load, and proceeding with the remainder by (**134**), (2nd), calculating for the unsymmetrical part by (**135**); and (3rd), then taking their algebraical sum of (1) and (2).

BOOMS.—From the web by (**123**).

139. Lattice Girders having their Diagonal Bars fixed together at their Intersection may be calculated as if the bars were not so fixed; for this mode of constructing a girder "can never add to the longitudinal strain upon it; but by calling into play the resistance of the bars to curvature adds to the stiffness of the bridge."†

140. Simple Truss.—Let $AB = l$, $WC = d$, and ϕ the angle between the inclined tie and the horizontal. Then,—

Fig. 54.



Compression in $AB = \frac{Wl}{4d}$,

Tension in AC and $CB = \frac{Wl}{4d} \sec. \phi$,

Compression on $CW = W$.

If WC represent $\frac{W}{2}$, then AW or BW will equal strain along AB; and AC or CB that in the ties AC, CB.

---

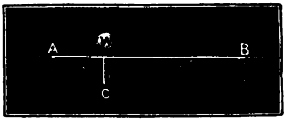
* The two methods will not give equal results at and about the very point (the centre of the girder) where the values are of the greatest consequence. This is quite unavoidable; and the safest course (which need be pursued only in works of importance) is to employ both methods, and to provide against the greater strains.

† "Wrought Iron Bridges," by J. H. Latham, M.A.

141. *When* W *is not in the Centre.*

Let $A\,W = a$,
$B\,W = b$,
$\phi =$ angle C A W,
$\phi' =$ angle C B W.
The other notations as before.

Fig. 55.



$$\text{Compression along } A\,B = W\,\frac{ab}{ld},$$

$$\text{Tension in } A\,C = W\,\frac{ab}{ld}\ \text{sect.}\ \phi,$$

$$\text{Tension in } C\,B = W\,\frac{ab}{ld}\ \text{sect.}\ \phi' \;.\;.\;.\;.\; (\mathbf{126},)$$

$$\text{Compression on } W\,C = W.$$

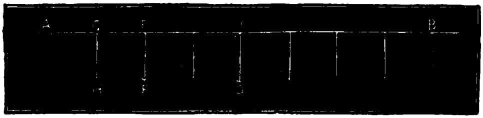
For the pressures on the supports, see (**117**).

142. *If the truss in either fig.* 54 *or fig.* 55 *were inverted*, the strains on the various parts would change in kind but not in amount.

143. In both the above cases, the tension on the ties and the direct (**87**) compression on A B would remain the same as at present, if instead of being loaded with the concentrated weight W, a load equal to (2 W) were spread uniformly along A B.

144. Compound Truss.—A structure of this kind may be regarded as compounded of a number of simple trusses (**140**). The simple truss A C B D has to sustain at its centre (C) half the entire weight of the structure and its load. The truss A E C F has to sustain at E half the total load between A and C. A G E H must likewise be considered as supporting at G half the total load between A and E, and so on with the others. With a uniformly distributed load on A B, A H E and the three other similar trusses would each take $\frac{1}{8}$; A F C and its fellow, each $\frac{1}{4}$; and A D B, $\frac{1}{2}$ of the total load. The strains on the various parts can then be obtained from (**140**). The compression along A B is uniform throughout the whole length.

Fig. 56.

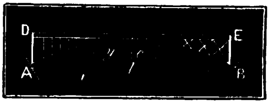


ARCH BRIDGES.

ARCHES WITH SPANDRIL BRACING.

145. With a Uniform Horizontal Load.

Fig. 57.



Let l = length of span,
v = rise, or versine,
y = horizontal distance of any point from the crown,
w = load per unit of length.

ARCHED RIB.—Compression at the crown (C),

$$C = \frac{wl^2}{8v}.$$

Compression at any other point (C′)

$$C' = \sqrt{\left(\frac{wl^2}{8v}\right)^2 + (wy)^2}.$$

The expression for C′ is strictly accurate only when the arch* is a parabola,—the curve of equilibrium for the load in question. It may, however, be safely used in most cases of practice for arcs of circles.

SPANDRIL.—*If the arch* be parabolic,* the only strain on the spandril will be the vertical pressure of the load. (See foot note †, p. 49.)

The spandril may then consist of a number of pillars or struts (as in fig. 57), each sustaining a vertical pressure $= \frac{wl}{N-1}$ nearly; N being the number of spaces into which the pillars divide the span.

If it consist of a continuous web, the compression of it per unit of length will be equal to w.

If the arch be not a parabola,* the strains in the spandril bracing may be obtained from **80**, **81**, (for continuous or "plate bracing"), or **111** (for diagonal open work), the rib being considered as the "compression boom."

TOP HORIZONTAL MEMBER.—*With a parabolic arch* this member (D E fig. 57) simply acts as an immediate support for the load (**87**). *With an arch not a parabola,* there will be a strain on it acting in the direction of its length, the nature and amount of which may be determined from the increments at the apices of the diagonals in the spandril, by the application of the law in **123** as illustrated in **134**.

146. With a Moving Load.

For obtaining the strains on the various parts of a braced arch, subject to a distributed moving load, the following method may be employed.†

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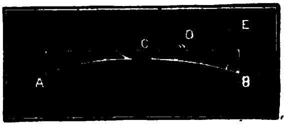
* More properly the neutral axis, or the line traced through the centres of gravity of the cross sections of the rib.

† Given by B. Stoney, B.A. (in "Theory of Strains"); and others.

Fig. 58.



Conceive the load to be collected into weights at the several apices on the horizontal member, each apex sustaining half the load on the two adjacent bays. The strains produced by each weight separately must then be found and tabulated, as in **135.** Suppose the strains from weight W (fig. 58) are to be considered. At W draw the vertical W D. From the abutment A draw A C D through the crown of the arch (C) till it intersects the perpendicular at D. Join D to B. If the weight W be resolved (**118**) in the directions A D and B D, the amounts and directions of the reactions of the two abutments will be found. This may be readily done by producing A D to E, and erecting a perpendicular at B. Then, if E B equal the weight W, D E will be the reaction of A, and D B the reaction of B. If the former be traced up from A towards the weight, and the latter from B, by the resolution of forces (**118, 119**) the strains on the various parts may be thus found; and the same being done for the other weights in succession, the maximum strains produced by any position of the load may be derived from a table similar to that on page 45.

If the weight of the structure be small compared with that of the rolling load, it will be found that some of the end bays of the horizontal member, and of the middle bays in the arch, will occasionally suffer tension. See also **137.**

UNBRACED ARCHES,

Or arches whose stability depends upon the stiffness of the rib itself.

147. The Neutral Surface, or neutral curve of an arched rib, is a line traced through the centres of gravity (**220**) of the cross sections of the rib.

148. The Line of Pressures *is a line the ordinates to which vary as the moments of rupture* (**1**) *for the load.*[*]

The line of pressures is given at once in those diagrams (pp. 4 to 6), in which the ordinates for the moments of rupture are directed to be taken from and on the same side of one horizontal line. Where this is not the case (as in **20** or **21**, for instance), the ordinates must be transferred to some new horizontal datum.

The ordinates may be taken to any scale for ready comparison with the neutral curve of the rib.

149. *Whenever in any arch the line of pressures coincides with the neutral surface, the arch is in equilibrium, and the strain upon it is everywhere compressive.*[†]

---

[*] **For Masonry** and many other **Arches** case **14** will be found exceedingly convenient. The structure and its load should be considered as made up of several small portions, each collected at its centre of gravity. The line of pressures can then be obtained by summing the ordinates for a new outline, as there directed.

[†] Strictly speaking, the pressure on an arch of other than a thoroughly incompressible material alters the form of the arch; and this alteration of form, or *bending* action, induces strains similar to those found in beams (see **1** and **23**). The tension on equilibriated ribs (**150, 151**) is in practice so small (even where it is developed,—for this does not occur till the tension induced by the *bending* exceeds the direct compression on the rib) that it need not be regarded.

150. The Stability of an Arch incapable of Resisting Tension is secure so long as the line of pressures does not at any point deviate from the neutral curve by an amount, the proportion of which to the depth of the rib is given by the values of q (**153**) for various forms of cross section.

151. Wherever the line of pressures deviates beyond this limit, there is a *tension on the rib* on the other side of the neutral surface, increasing with the deviation.

152. *Pressure along one edge* of an arched rib invariably produces tension on the other edge, whatever be the form of cross section.

153. Limits of the deviation of the line of pressures from the neutral surface consistent with there being no tension on the rib ;—

Form of Cross Section.	*Value of q.**
Rectangle	$\frac{1}{6}$
Ellipse and Circle	$\frac{1}{8}$
Hollow Rectangle (area = $b\,h - b'\,h'$) also **I** formed section, b' being the sum of the breadths of the lateral hollows.	$\frac{1}{6}\left(1 - \frac{b'\,h'^3}{b\,h^3}\right) \div \left(1 - \frac{b'\,h'}{b\,h}\right).$
Hollow Square (area = $h^2 - h'^2$) .	$\frac{1}{6}\left(1 + \frac{h'^2}{h^2}\right).$
Hollow Ellipse	$\frac{1}{8}\left(1 - \frac{b'\,h'^3}{b\,h^3}\right) \div \left(1 - \frac{b'\,h'}{b\,h}\right).$
Hollow Circle	$\frac{1}{8}\left(1 + \frac{h'^2}{h^2}\right).$

b and b' = external and internal breadths ; and

h and h' = the external and internal heights or depths.

I, section alike above and below .	$\frac{1}{6}\left(1 + \frac{4\,A}{A' + 2\,A}\right).$

A = area of each table or flange ;

A′ = area of the connecting web.

154. The above values of q should be applied as a test for tension in cases of braced arches, and also where the spandril consists of columns ; for the line of pressures is then in reality a polygon, with the angles at the apices on the rib. (See fig. 59.)

Fig. 59.

154 A. When an **Arched Bridge** consists of **several** unequal **Spans**, the neutral surfaces of all the ribs should be parts of the same figure—(generally arcs of the same circle, or the same parabola).

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* These values of q are from Professor Rankine's "Civil Engineering."

SUSPENSION BRIDGES.

155. ORDINARY SUSPENSION BRIDGE OF ONE SPAN, WITH A UNIFORM HORIZONTAL LOAD.

MAIN CHAINS.—The curve which the main chains will assume will be very nearly a parabola.

Let w = load on each chain per unit of length;
l = length of span;
v = versine, or depression of the chain;
ϕ = angle which a tangent to the chain at any point makes with a horizontal;
y = horizontal distance of any point from the mid-span.

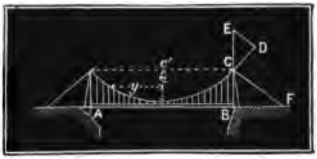
Tension at centre, $T = \frac{w l^2}{8 v}$;

At any other point,

$$T' = \left(\frac{w l^2}{8 v}\right) \sec. \phi.$$

$$\text{Or, } = \sqrt{\left(\frac{w l^2}{8 v}\right)^2 + w y)^2}.$$

Fig. 60.



SUSPENDING RODS.—Let N = the number of spaces into which they divide the span, then,

$$\text{Tension on each} = \frac{w l}{(N - 1)}.$$

TOWERS AND COUNTER-CHAINS.—The tension on the counter-chains, and the pressures on the towers may be readily found as follows:—Produce the tangent to the main-chain at the tower (C) till its length (C D) on a scale of parts, equals the tension at that point (found from T' above). Through D draw D E parallel to the direction of the counter-chain (C F). Produce the centre line of the tower till it intersects D E. Then (**118**) C E will give the pressure on the tower, and D E the tension on the counter-chain.

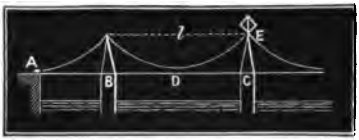
156. ORDINARY SUSPENSION BRIDGE OF MORE THAN ONE SPAN, WITH A UNIFORM HORIZONTAL LOAD.

When a suspension bridge consists of several spans, the chains of all of them must form portions of one and the same parabola.

The strains on the whole spans—as B C—will be the same as in (**155**).

The strains on the outer spans (A B) are identical with those in B D, as the parts A B and B D are equal. But where the abutment spans are not equal to half the inner ones, the first conditions made in this paragraph must be observed, and then $T' = \left(\frac{w\,l^2}{8\,v}\right)$ sec. θ (**155**), l being still the inner span.

Fig. 61.

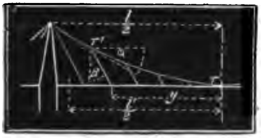


The compression on the towers will equal the diagonal of a parallelogram, whose sides correspond in direction and length to the tensions in the chains on either side (as at E in fig. 61).

157. SUSPENSION BRIDGE WITH SLOPING RODS, WITH A UNIFORM HORIZONTAL LOAD.

MAIN-CHAINS.—The curve which each half of the chains will assume will be a parabola with its axis parallel to the direction of the sloping rods.

Fig. 62.



Tension at mid-span, $T = \frac{w\,l'\,l}{8\,v}$; where ($w\,l'$) is the total vertical load on the rods; the other notations as in **155**.

Tension at any other point,

$$T' = w\,y \text{ cosec. } \phi\,;$$

y being the distance from the mid-span to the bottom of the sloping rod, at the top of which T′ is required; and ϕ the angle which a tangent to the chain makes with the horizontal.

SLOPING RODS.—Tension on any rod,

$$t = W \text{ sec. } \beta\,;$$

where W is the vertical load on the rod (half the load on the two adjacent bays); and β the angle which the rods make with a vertical.

HORIZONTAL MEMBER.—The compression (c) on the horizontal platform at any point distant (y) from the mid-span, is

$$c = w\,y \text{ tan. } \beta.$$

For the *towers*, see **155** or **156**.

158. SUSPENSION BRIDGES WITH MOVING LOADS are subject to much disfigurement, to prevent or modify which several means have been devised.

159. (1.) *An auxiliary girder* from pier to pier, anchored down to the abutments. If this girder be continuous for each span, its booms (fig. 38 A) for about the middle half of its length must be able to resist a strain $= \frac{w'\,l^2}{24\,d}$, and the web a shearing force of about $\frac{w'\,l}{3}$; w' being the intensity of the moving load per unit of length; l, the length of span, and d the depth (**98**) of the girder.

160. (2.) When the bridge consists of several spans,—*Fixing the chains to the top of the towers*, and considering the latter as semi-girders, each one loaded at the extremity with $\frac{w' l^2}{8 v}$, notation as before (**159**). While the tower is suffering the strains consequent on the application of this force to its extremity, there is also a direct compression on it $= \left(w + \frac{w'}{2}\right) l$. This latter, it must be remembered, modifies the tensional (4) strains produced by the former.

161. (3.) *Inserting diagonal bracing* between the roadway and the chains. The strains on the various parts may in this case be obtained as in **146**; they will be altered in kind only, not in intensity.

161 A. (4.) *A pair of chains of identical curvature*, placed one above the other, and having diagonal bracing between, the greatest shearing force on which would be $\frac{2 w' l}{7}$.

161 B. (5.) *Counter chains attached to the main chains* at about $\frac{1}{4}$ span from the abutments or piers, and running down to the latter. They should be made to resist a strain of $\left(\frac{w' l}{2} \times \sec. \theta\right)$; θ being the angle between the counter chains and a vertical.

161 C. (6.) *Inclined straight chains*, for carrying the platform and the moving load. They extend from the towers, and meet or intersect each other. They are sustained in the required straight lines by rods, which are connected to curved chains, the latter having to carry the weight of the straight chains only.* The tension on the latter may be found from **155**. The tension on the straight chains may be most readily found by a parallelogram of forces (**118**).

162. Abutments and Piers. *Girders*, properly so called, viz., those structures which simply *rest upon* the supports, bring upon those supports a vertical pressure equal to the shearing force developed there. (See last paragraph in **46**.)

Abutments of Arches. The thrust at the abutments of an arch is exactly equal to the compression in the arch rib at the springing, the value of which may be determined from **145, 146.**

For the towers and piers of suspension bridges, see **155, 156.**

Whenever the piers of a bridge consist of columns, their strength *as such*—their liability to flexure, &c.—must not be overlooked. (See **168.**)

SECTION III.

DISTRIBUTION OF MATERIAL TO RESIST THE CALCULATED STRAINS.

(Embodying Processeses VI, VII, p. 24.)

163. The Strength of a Structure, or of any part of it, is its ability to resist the external forces tending to cause its rupture.

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* Mr. R. M. Ordish's system.

164. Axiom. *No whole is stronger than its weakest part.*

165. Uniform Strength. A structure is said to be of uniform strength when no one part would yield before another, supposing the structure to be subjected to the load, or a multiple of that load for which it was designed.

In structures not of uniform strength, all the material in excess of that necessary for uniformity of strength is redundant.*

To secure uniformity of strength, a constant co efficient of safety (**172**) must be used for the same material strained in the same way. For beams of uniform strength, see **215—218.**

166. Units. It is necessary to adopt:

1. Unit of Strain or Stress; generally 1 lb. avoirdupois.
2. Unit of Sectional Area; generally 1 superficial inch.
3. Compounded Unit of Strain and Area; 1 lb. per sq. inch.

Let A = area of a section in units (sq. inches).
S = calculated strain in units (pounds avoirdupois).
U = ultimate strength, or breaking weight of the material—in lbs. per sq. inch of section. (For numerical values, see **231.**)
Co = coefficient of safety (**172**).
W S = working strain (**172**).

PRINCIPAL STRAINS TO BE MET WITH IN BRIDGES, GIRDERS, &C.

167. Tension, causes or tends to cause the fracture of the material upon which it acts by tearing asunder its particles.

The resistance to Tension is directly as the area of the cross section of the material, taken perpendicular to the direction of the strain (**164**).

Area necessary to safely resist a strain, $A = \frac{S \times Co}{U} = \frac{S}{WS}.$

168. Compression, causes or tends to cause the failure of the material, by crushing, buckling, or both combined.

Crushing. Materials in compression ("struts") can be considered liable to crushing alone, only when their least diameter (taken perpendicular to the direction of the strain) is not more than about ⅕ of their unsupported length,† The resistance is then directly as the sectional area.‡

Area necessary to safely resist a strain, $A = \frac{S \times Co}{U} = \frac{S}{WS}.$

Buckling. When struts have an unsupported length equal to about 25§ times their least diameter, they may be considered to suffer almost entirely by being buckled, *i. e.*, crumpled up.[*] Struts shorter than this fail, partly by crushing, and partly by buckling.

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* It cannot be said that the excess is entirely useless; but more on this point would be out of place here.

† Hodgkinson.

‡ The resistance to crushing of a body whose diameter normal to the pressure far exceeds its dimensions in a line with the pressure, is very great, but equally indefinite.

§ For wrought-iron struts with riveted joints, from 40 to 50 times.

169. BREAKING WEIGHT OF COLUMNS.—*Prof. Hodgkinson's formulæ for cast-iron columns.* When of more than from 25 to 30 diameters in length,

$$\text{Break. wt.} = \frac{44{\cdot}16\, d^{3\cdot6}}{l^{1\cdot7}} \text{ for solid pillars ; and}$$

$$\text{Break. wt.} = \frac{44{\cdot}3\,(d^{3\cdot6} - d'^{3\cdot6})}{l^{1\cdot7}};$$

for hollow cylindrical pillars, flat, or firmly fixed at both ends, d being the external diameter, and d' the internal diameter in inches, and l the length in *feet.* Columns rounded or moveable at both ends have but ⅓ the strength of those flat or fixed ; and the strength of those with one end fixed or flat, and the other rounded or moveable, is about an arithmetical mean between these two cases.

When of less than from 25 to 30 diameters in length, let b be the value obtained by the above formulæ, and c the crushing load of a short block (**231**) of the same sectional area as the column, then the corrected breaking weight $= \dfrac{b\,c}{l + \frac{3}{4}c}$.

General formula for the breaking weight of Cast and Wrought Iron Columns.

Let C = compressive resistance of a short block of the same sectional area.

$$r = \frac{l}{d} = \text{length of column divided by the greatest diameter.}$$

For cast-iron, $\quad \text{B. W.} = \dfrac{C}{{\cdot}68 + {\cdot}1\,r}.$

For wrought iron, $\quad \text{B. W.} = \dfrac{C}{{\cdot}85 + {\cdot}04\,r}.$

Breaking Weight in tons per square inch of section.

$$\text{Round}\begin{cases}\text{cast iron,} & \dfrac{13{,}500}{330 + r^2}.\\[2ex] \text{wrought iron,} & \dfrac{34{,}000}{2000 + r^2}.\end{cases}$$

$$\text{Square timber,} \qquad \frac{850}{350 + r^2}.$$

Breaking Weight of timber pillars, taking the strength of a cube as unity.

Value of r . . .	1	12	24	36	48	60
Breaking weight . .	1	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{12}$

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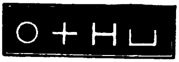
[*] Professor Hodgkinson.

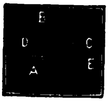
170. LONG STRUTS should be made with a cross section, which will insure a certain amount of rigidity or stiffness, and thus resist the tendency to buckle: or they should be braced (either externally or internally), and thus divided into a number of shorter lengths, each of which (and, therefore, the braced strut as a whole) may be considered as suffering crushing alone.

Fig. 63.



171. Shearing causes, or tends to cause, contiguous sections of the material to slide over each other (as at A B, supposing that D E is part of a loaded beam, A one support, and B C part of the load). The resistance to shearing is directly as the area that would be sheared.

Fig. 64.



Area necessary to safely resist a shearing strain,

$$A = \frac{S \times Co}{U} = \frac{S}{W\,S}.$$

Shearing strains will be found to act on the vertical web * of continuous webbed girders, and in joints generally, which see.

For *Bending, bending intensities, and the resistance of materials to them*, see **1**, and **191** et seq.

172. Coefficients of Safety are numbers representing the proportions of the ultimate strength of materials to the strains that can safely be brought upon them. Coefficients of safety may be variously estimated. The following may, however, be taken as a fair average of the factors at present in use where the materials employed and the workmanship are ordinarily good.

	Metals.	Timber.	Masonry.
For a dead load . .	3	4 to 5	4
For a live load . .	5	8 to 10	8

Under *dead load* may be included all permanent or stationary loads, and loads very gradually applied.

Under *live load*, all rapidly moving, and suddenly applied loads.

$$\frac{\text{Ultimate resistance}}{\text{Coefficient of safety}} = \text{Working strain.}$$

173. The **Modulus of Elasticity** (E) (in pounds per square inch as given in **231**) is the weight (in pounds) required to elongate or shorten, by an amount equal to its original length, a bar of material (of one square inch of cross-section), and is on the supposition that the elasticity of the material would remain constant throughout the operation.

$$\frac{\text{Strain (in lbs.) per sq. inch on a bar}}{\text{Mod. of elas. (in lbs.) per sq. inch}} = \frac{\text{Increase or diminution of length}}{\text{Original length of the bar}}.$$

JOINTS.

174. *Joints should always be as strong as the parts they serve to join.*

---

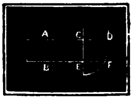
* In the paragraphs **62** and **74**, the flanges have not been supposed to take part in resisting the shearing forces (so called from the shearing strains above), any more than the web in taking its share of the horizontal strains. (**66**.)

175. *The various parts of a joint, and the several parts in a fastening, should be of uniform strength.*

IRON JOINTS AND FASTENINGS.

176. Rivets may fail in several different ways, depending on their office in a joint. The head may be shorn off (as at C D, E F); or the rivet may be ruptured at any section (as A B) when the rivet is in tension.

Fig. 65.



Let S = the tensional strain; then d (diam. of rivet) should not be less than $\sqrt{\frac{S \times Co}{U \times \cdot 7854}}$, U being the ultimate resistance of the material to tension (**231**); and C D or E F should not be less than $\frac{S \times Co}{U \times 3 \cdot 1416\, d}$ (**172**), U being here the ultimate resistance to shearing (**231**).

When the rivet has to resist a shearing strain (S) at A B, d must not be less than $\sqrt{\frac{S. \times Co}{U \times \cdot 7854}}$, U being the ultimate shearing resistance.

General Rule.—The height of the head (h) *should never be less than half the diameter of the rivet.*

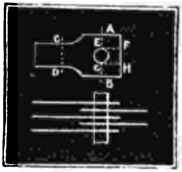
177. Bolts.—The diameter of a bolt liable to shearing at the spindle (as at A B, fig. 65) must be determined from (**176**).

If the bolt be in tension it will fail either—1st, by shearing off the thread; 2nd, or by shearing off the head; 3rd, or by tensional rupture of the spindle. For a perfect thread the height of the nut and of the head should be equal: but to allow for inaccuracies of workmanship, the height of the nut should be about twice that of the head. The height of the nut should not be less than the diameter of the spindle; in practice it is frequently made much more than this.

178. *The diameter of a nut or head of a bolt, or of the head of a rivet, should be not much less than twice that of the spindle.*

179. Pin Joints in Tension Bars.—(Such as in some suspension chains, triangular girders, and trusses).

Fig. 66.



Pin.—Let n = the least number of sections at which the pin must be divided before the joint can fail (4 in fig. 66), a = sectional area of pin, and S the tension on the joint, then $a = \frac{(S \times \text{Coef. of safety})}{(\text{Ult. shear.}) \times n}$. The coefficient (**172**) should be large, as any inaccuracies in the workmanship will tend to concentrate the strain in certain parts.

Link.—The section of any link-head taken through the centre of the pin-hole (A B) should equal about half as much again as that taken through the body of the link (as C D), in consequence of the inequality in the distribution of the strain.

Let l = the length of overlap, E F or G H (fig. 66); t = thickness of all the overlaps, in one series, taken together (2 or 3, fig. 66), then

$$l = \frac{S \times Co}{2 \times U \times t},$$

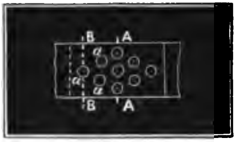
where S is the strain on the joint, and U the ultimate resistance to shearing.

General Rule.—Diameter of pin may be ¾ of the width of the links.

Riveted Joints in Tension.

180. The Effective or Available Section of a plate with rivet holes in it depends upon the disposition of the rivets. Thus, in fig. 67, the least section of *plate* that could be taken is that at A A. But before the joint can fail here by the rupture of (say) the upper plate at A A, the three rivets marked *a a a* must be shorn. And it will be found that in a joint arranged in this way, the effective section will be equal to that *taken through the first rivet, or line of rivets*, as at B B.

Fig. 67.

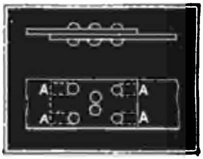


181. *The sectional area of all the rivets in a joint taken together should be equal to the effective section of the plate.*

The distance between the centres of rivets which stand in a line (perpendicular to the tension on the joint) should be made $= d + \frac{1}{t}(\cdot 7854\, d^2 n)$, in which n is the number of lines of rivets as above (5 in fig. 67); t, the thickness of the plate; and d diameter of rivets.

182. Lap Joints may fail,—1st, from the tensional rupture of the effective section (**180**) of the plate; 2nd, by the shearing of the rivets; 3rd, by the shearing out of the overlaps (A A A A, fig. 68). *The strain on each rivet* $= \frac{\text{Str. on joint}}{\text{Num. of riv.}}$, and they have each to be shorn at one section only (**179**). The distance between the lines of rivets (**181**), (of which there are 3 in fig. 68) must not be less than the overlap required for the rivets in the first row (as at A). The latter may be determined from the following equation,

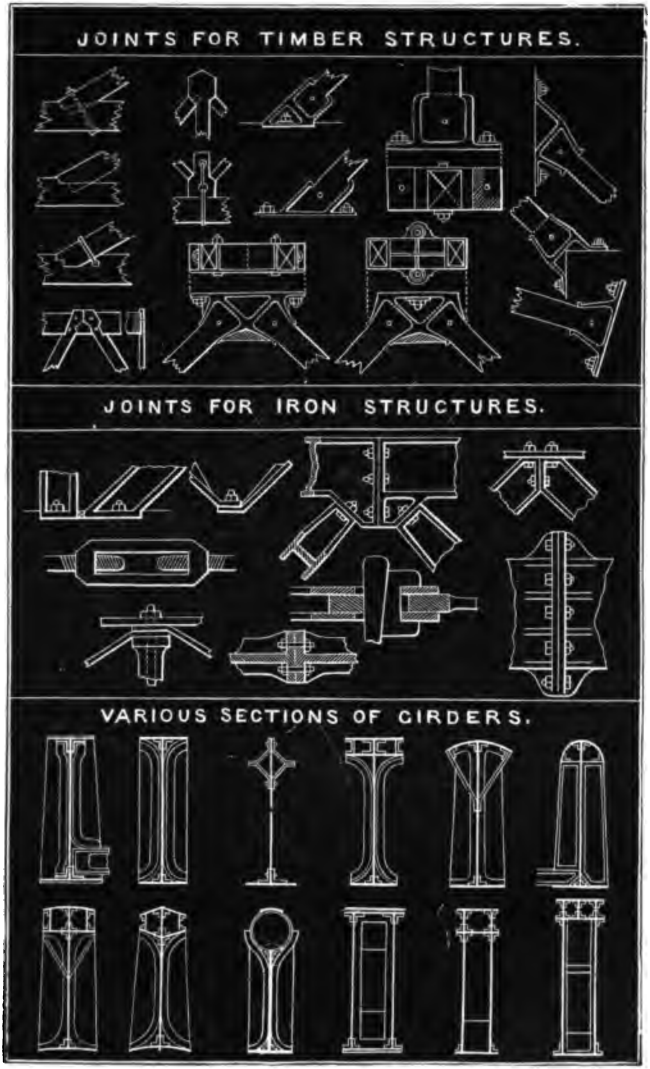
Fig. 68.



$$l = \frac{S \times Co}{2\, n\, t\, U},$$

in which S = tension on the whole joint; n, total number of rivets; t, thickness of plate; U, ultimate resistance of the plate to shearing; Co, a suitable coefficient of safety (**172**).

PLATE II.


JOINTS FOR TIMBER STRUCTURES.
JOINTS FOR IRON STRUCTURES.
VARIOUS SECTIONS OF GIRDERS.

183. Fish Joints.—Where only one cover or fish-plate is used (as in fig. 69), the case is virtually identical with two successive lap-joints, and can be calculated as such (**182**). Where two cover-plates are employed, it is to be borne in mind, that before the joint can fail, each rivet must be shorn at *two* sections, so that the section of each need be but half that necessary with a single cover-plate (**164**). The thickness of each cover-plate must never be less than half that of the main plates.

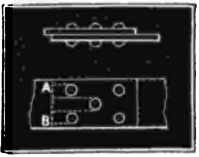
Fig. 69.

Riveted Joints in Compression.

184. In **lap-joints** the effective sectional area of the plate is equal to the loss of section by rivet-holes, not counting those which are behind any others in the direction of the pressure. Thus, in fig. 70, the effective bearing section of the plate is that from A to B.

For the shearing strain (**171**) on each rivet (**176**), divide the strain on the joint by the total number of rivets. A rather large coefficient of safety (**172**) should be used, as inaccuracies of workmanship will materially concentrate the strain at certain points.

Fig. 70.



185. Butt Joints (same as fish-joints in tension) may be considered as having an effective section equal to the total section of the plate; for if the rivets fill the holes as they should, there is hardly any loss of strength from them.

The cover-plates are required simply to keep the main plates in their proper positions.

186. Joints formed by **gibs and cotters** may be calculated from **179**. It is advisable that the obliquity of the surfaces to the direction of the strain should not exceed 4° or 5°.

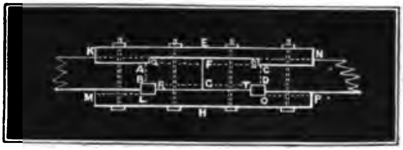
A few cast and wrought iron joints are given in Plate II.

JOINTS IN TIMBER STRUCTURES.

187. A form of joint having been determined, the parts at which it is liable to fail must be traced out, and sufficient strength be given to them, using (**165**—**171**), and particularly observing (**164**).

188. Joints in Tension.—*Fished and scarfed joints.* Figs. 71, 72 repesent forms of fish-joints. In fig. 71 the tension may cause rupture at A B, or C D; or at E F *and* G H. (*Both* of the latter must fail before the joint gives way by the rupture of the fish-pieces.) And further, the joint may fail by the shearing off of Q F and R G; or of F S and G T; or of K Q and L M; or of S N and O P. The

Fig. 71.



above is disregarding the shearing resistances of the four bolts shown in fig. 71, which must be taken into account in a somewhat similar way.

Fig. 72.

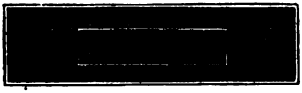


Fig. 72 represents a joint fished with iron plates, and also "scarfed." Fig. 73 shows another form of scarfing.

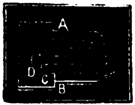
Fig. 73.

189. Joints in Compression.—Surfaces abutting against each other should be as nearly as possible perpendicular to the direction of the thrust. Where convenient, fish pieces may be used to retain the main-pieces in their proper position; or the latter may abut into cast-iron sockets suitably designed.

On Plate II. are given several forms of joints for timber structures, which can profess only to be suggestive.

190. Shouldered Tenon for attaching Cross to Main Beams.—The weight on the end of the cross beam AB is borne by the shoulder C, which is let into the main beam for a distance equal to about one-sixth of the depth of the former. The length of the tenon, D, is about twice its depth.

Fig. 74.



BEAMS OF VARIOUS SECTIONS.

191. For those beams, girders, and other similar structures, in which certain parts are supposed to resist certain definite strains (**62**), and other parts, other strains (for instance, flanged girders with *thin* continuous webs, all open-webbed girders, trusses, &c.) see **63** to **162**. A mode of procedure is there adopted which would not be thoroughly applicable to those beams in which every fibre or particle is considered to take part in resisting the bending action of the moment of rupture (**1**), and where the whole section is liable to the action of the shearing force (**1**).

In Designing a Beam—

192. Determine the nature of its cross section.

If the exact proportions of the section are to be adhered to, and the area alone required,—express all the dimensions of the section in terms of one of them, that there may be but one unknown quantity.

Thus—suppose a beam to support a given load is to be rectangular, with the depth twice the breadth, then let b = breadth, and $2\,b$ = depth.

If all the dimensions of the section, except one, be given, that one will of course be the unknown quantity. Then,—

193. Substitute for M, in the equations given hereafter, its value as found from the span, manner of loading and supporting, &c., pp. 2 to 15. The dimension, or dimensions, required may then be obtained. Lastly,—

194. If at any vertical section, there be not sufficient material to resist the shearing force (**1**, **34**, *et seq.* and **171**), the necessary addition must be made to the section.

This will seldom be required at other places than near the supports in discontinuous beams, and near the points of contrary flexure (**23**) in continuous beams.

195. *The weight of the beam itself* must always* be added to the extraneous load upon it; and may be approximately estimated by a process similar to that in **60**.

---

196. THE STABILITY OF A LOADED BEAM depends on the equation

$$M = R. \quad (1.)$$

197. Abbreviations—

Let M = moment of rupture (**1**, **2**), the values of which may be determined from the several cases **7** to **33**.

$R = \dfrac{C\,I}{t}$ = moment of resistance (**1**) of the section.

I = moment of inertia of the section.

t = distance of the neutral axis (**198**) from the farthest edge of the section.

a = total area of the section.

C = modulus of rupture (**203**).

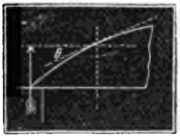
198. The Neutral Axis (N — A in the sections, figs. 82—84) is a section of the neutral surface,—a layer in the beam (and the only one) which is neither extended nor shortened by the action of the load (**4**).

199. Provided that the limits of elasticity of the material of the beam be not exceeded, *the neutral axis will pass through the centre of gravity of the section* (**220**).

200. There must be but one lineal unit used in obtaining the values of M, R, I, t; and the superficial unit used for a must correspond to that lineal unit.

201. The section at which R and I (**197**) are taken may be made parallel to the reaction of the supports of the beam.†

Fig. 75.



202. Wherever either the upper or lower surface of the beam is not perpendicular to the action of the load, then C must be modified to ($C \cos.^2 \theta$), θ being the inclination of the most inclined surface to that perpendicular.

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* Except in small girders, or beams of minor importance.

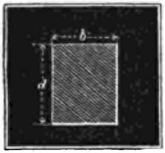
† The section at which the moments of resistance should really be taken is a curved surface cutting the upper and lower edges of the beam, and the neutral surface all at right angles; whatever be the form of the beam; and the moment of rupture to be equated with it should be taken at the intersection of this curved surface with the neutral surface of the beam.

203. Modulus of Rupture.—The *theoretical* value of C is the resistance of the material to direct compression or tension. But it is found from experiments on cross breaking that this value is not sufficiently high. Amongst the reasons that have been assigned for this, are—1st, that in addition to the resistances of the particles of the beam to a direct strain, there is another resistance arising from the lateral adhesion of the fibres to each other, termed the "Resistance of Flexure." (See Barlow on the "Strength of Materials," 6th edition.*) And 2nd, that in most metallic beams (especially when cast) the outer skin, which is strained more than any other part of the section, is very much stronger (from many well-known causes) than the average section; whereas if the direct tensile or compressive resistance of the same beam, in the direction of its length, were being experimentally ascertained, it would be the average section at least, and perhaps the centre (weaker) portion especially, from which the strength would be determined. However, there is evidently a necessity to employ a higher value than that for the direct resistance; and Professor Rankine has adopted a modulus of rupture which is 18 times the load required to break a bar of 1 sq. inch section, supported on two points one foot apart, and loaded in the middle between the supports (**231**).

MOMENTS OF INERTIA AND RESISTANCE OF BEAMS OF VARIOUS SECTIONS.

204. Beam of a solid rectangular section.

Fig. 76.

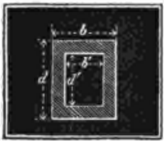


$$I = \frac{b\,d^3}{12} = \frac{a\,d^2}{12}.$$

$$R = \frac{C\,b\,d^2}{6} = \frac{C\,a\,d}{6} = M.$$

205. Beam of a hollow rectangular section.

Fig. 77.

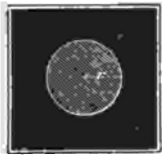


$$I = \frac{b\,d^3 - b'\,d'^3}{12}.$$

$$R = \frac{C\,(b\,d^3 - b'\,d'^3)}{6\,d} = M.$$

206. Beam of a solid circular section.

Fig. 78.



$$I = \cdot 7854\,r^4 = \frac{a\,r^2}{4}.$$

$$R = C\,\cdot 7854\,r^3 = \frac{C\,a\,r}{4} = M.$$

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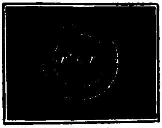
* Edited by W. Humber. London: Lockwood & Co.

207. Beam of a hollow circular section.

Fig. 79.

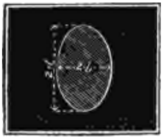


$$I = \cdot7854\,(r^4 - r'^4).$$

$$R = \frac{\cdot7854\,C\,(r^4 - r'^4)}{r} = M.$$

208. Beam of a solid elliptical section.

Fig. 80.

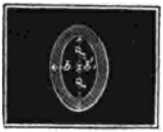


$$I = \cdot7854\,b\,d^3.$$

$$R = \cdot7854\,C\,b\,d^2 = M.$$

209. Beam of a hollow elliptical section.

Fig. 81.

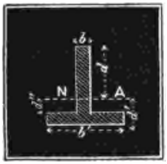


$$I = \cdot7854\,(b\,d^3 - b'\,d'^3).$$

$$R = \frac{\cdot7854\,C\,(b\,d^3 - b'\,d'^3)}{d} = M.$$

210. Beam with one flange.

Fig. 82.

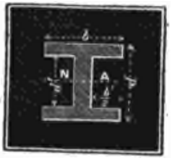


$$I = \tfrac{1}{3}\left\{ b\,d^3 + b'\,d'^3 - (b' - b)\,d''^3 \right\}.$$

$$R = \frac{C\,I}{t} = M.$$

211. Beam with two equal flanges.

Fig. 83.

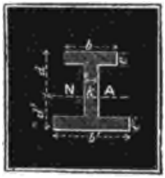


$$I = \frac{b\,d^3 - b'\,d'^3}{12}.$$

$$R = \frac{C\,(b\,d^3 - b'\,d'^3)}{6\,d} = M.$$

212. Beam with two unequal flanges.

Fig. 84.



$$I = \tfrac{1}{3}\left\{ b\,d^3 - (b - k)\,(d - c)^3 + b\,d'^3 - (b'-k)\,(d' - c')^3 \right\}.$$

$$R = \frac{C\,I}{t} = M.$$

213. To find the Moments of Inertia and Resistance of any Cross Section made up of a number of simple figures.

Find the moment of inertia of each of the simple figures about an axis traversing its centre of gravity parallel to the neutral axis of the complex figure.

Multiply the area of each of the simple figures by the square of the distance between its centre of gravity and the neutral axis of the whole figure.

Add all the results together for the moment of inertia of the whole figure.

Let I_1 = moment of inertia of one of the simple figures about its own neutral axis; A its area; v the distance from its centre of gravity to that of the whole section (**220**); and I, moment of inertia of the whole section; then,

$$I = (I_1 + v^2 A) + \&c.$$

$$\text{Moment of resistance, } R = \frac{C\,I}{t} \quad . \quad . \quad . \quad . \qquad (\mathbf{197}.)$$

214. Moments of Inertia and Resistance, Ultimate Strength, and Deflections of Similar Beams.

Moments of inertia	of similar sections vary as	the 4th power	of their linear dimensions.
Moments of resistance		the 3rd power	
Strengths		the 2nd power	
Deflections		the 1st power	

BEAMS OF RECTANGULAR SECTION AND OF UNIFORM STRENGTH (165).

Elevations of Beams of Constant Breadth, and Plans of Beams of Constant Depth.

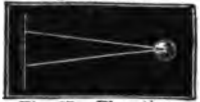
215. A semi-beam (7) loaded with a concentrated weight at its extremity.

A B, a parabola, with its vertex at B.

Fig. 85.—Elevation.

A triangle.

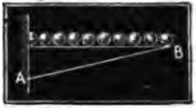
Fig. 86.—Plan.



216. Semi-beam (9) loaded uniformly over its entire length.

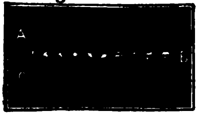
A B, a straight line.

Fig. 87.—Elevation.



A B and C B, a pair of parabolas, with their apices at B.

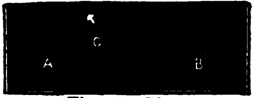
Fig. 88.—Plan.



217. Beam supported at both ends, loaded at any point (13) with a concentrated weight.

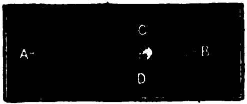
A C and B C, pair of parabolas, with their vertices at A and B respectively.*

Fig. 89.—Elevation.



A C D, B C D, a pair of triangles, having a common base, C D.*

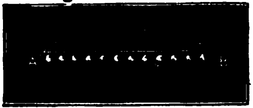
Fig. 90.—Plan.



218. Beam supported at both ends, loaded uniformly (19) over its entire length.

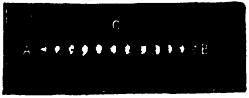
A semi-ellipse, A B major axis.*

Fig. 91.—Elevation.



A C B, A D B, pair of parabolas, having their vertices at mid-span.*

Fig. 92.—Plan.



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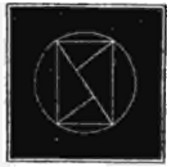
* The additional material dotted in at the supports is necessary to resist the shearing force (194).

219. How to cut the, (1,) Strongest and, (2,) Stiffest Rectangular Beam from a Cylindrical Log.

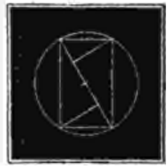
Let the accompanying figs. 93, 94 represent sections of the log.

Fig. 93.



Strongest.

Fig. 94.



Stiffest.

Draw a diameter. For the beam whose ultimate strength will be the greatest, trisect the diameter. For the beam which will deflect the least, divide the diameter into four equal parts.

Draw perpendiculars to the diameter as shown, and their intersection with the circumference will determine the inscribed rectangle, which is the section of the required beam.

220. To find the Centre of Gravity (199) of any Section.

Let a, a', a'', &c., represent the sectional area of the several elementary parts into which the section may be decomposed; g, g', g'', &c., the known distances of their respective centres of gravity from any fixed axis—say the lower edge of the beam—and S, the distance from the latter to the centre of gravity of the total section; then,

$$S = \frac{a\,g + a'\,g' + a''g'' + \&c.}{a + a' + a'' + \&c.}.$$

DEFLECTION.

221. Deflection is the "displacement of any point of a loaded beam from its position when the beam is unloaded."

222. Camber is an upward curvature, similar and equal to the maximum calculated deflection, given to a beam or girder or some line in it, in order to ensure its horizontality when fully loaded.

GIRDERS WHOSE CROSS SECTIONS ARE UNIFORM AND EQUAL THROUGHOUT THEIR LENGTHS.

223. The maximum deflections for several cases are given with the moments of rupture (7 to 25), by the values for Def.

FLANGED GIRDERS OF UNIFORM STRENGTH.

224. Girder supported at both ends.

Let D = central deflection.

d = central depth.

l = length of span.

K = sum of the extension of one flange or boom, and the shortening of the other by the strains upon them.

$$\text{Then } D = \frac{K\,l}{8\,d}.$$

K may be found as follows: Let S = strain in lbs. per sq. in. on either boom when the load producing the deflection is on the beam; E = modulus of elasticity (**173**); l_1 = length of boom; then $k = \pm \frac{S\,l_1}{E}$ = extension or compression in length of boom after the strain is on; and if k' correspond similarly for the other boom, then $k + k' = K$. If the booms be of equal length and section, then $K = (2\,k)$.

225. Semi-girder.

Let d = depth at support; the other notations as before.

The deflection at the unsupported extremity,—

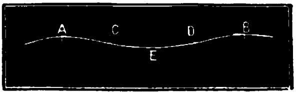
$$D = \frac{K\,l}{2\,d}.$$

226. Continuous Girders, and Girders fixed at One or Both Ends (**24—33**).

When any whole span is analysed it will be seen (**24** to **30**) that it is equivalent to a whole girder supported at the ends, and one or two semi-girders, as the case may be. To these the above formulæ (**223—5**) may be applied, and the maximum deflection obtained. Thus—

For *a girder* (as A B) *fixed at both ends* (**24—26**), the deflection of the semi-beams A C and D B at C and D (**11** or **225**), added to the deflection of C D below its ends, as obtained from (**12**, **19**, or **224**), will be the total maximum deflection of E below A. B.

Fig. 95.



Again, for *continuous girders with moving loads* (**32**, **33**), *the maximum deflection* at the middle of a span (B C, fig. 25) *will occur simultaneously with the maximum positive moment of rupture* (M_x, page 14), at which time the points of contrary flexure will be at M, M_1 (fig. 25), whose positions may be determined either from the diagram, or formulæ. *In the outer spans of continuous girders, and in girders fixed at one end and supported only at the other*, the deflection at the middle of the part corresponding to a whole girder simply supported (see **28**, **29**), may be found by adding the central deflection of the latter as such (**19**, **224**) to half the deflection of the remaining (semi-beam) portion (**11** or **225**).

BREAKING AND SAFE LOADS FOR BRIDGES, GIRDERS, ETC.

227. In the whole of the foregoing pages it is supposed that the span, load, and other data as far as necessary are given, in order to find the resulting strains, and the quantity of material to resist them.

228. To find the Load (breaking or safe) **when the Quantity of Material, Span, &c., are given,** is simply an inversion of the former calculations.

For sectional area of material, substitute an equivalent strain, breaking or safe according as breaking or safe load is required.

Then ascertain the load that would produce that strain, and that will be the load required.

Remembering that if the structure be not of uniform strength, the strength of the weakest part determines the strength of the whole (**164**).

229. *Example* I.—What load, at the centre of a wrought-iron, single-webbed or plate girder of uniform strength (**165**), and of the following dimensions, would cause the rupture of the girder,—Length of span, 20 ft.; central depth (**67, 75**), 1·5 ft.; effective section (**180**) of the lower flange at the centre, 4 sq. in.; iron of average quality (**231**).

As the girder is of uniform strength, it matters not what part is considered, for the same result would be obtained.

Lower flange will be in tension (**65—1**).

Ultimate strength of average wrought iron plate (**231**), 55,000 lbs. per sq. in.

$$\overset{\text{sq. in.}}{4} \times \overset{\text{lbs.}}{55{,}000} = \overset{\text{lbs.}}{220{,}000} = \text{breaking strength of lower flange.}$$

$$\text{Then from (12 and 67) strain on lower flange} = \frac{W\,l}{4\,d} = 220{,}000 \text{ lb.} = \frac{W \times 20}{4 \times 1{\cdot}5}.$$

$$\text{Therefore } W = \frac{220{,}000 \times 4 \times 1{\cdot}5}{20} = 66{,}000 \text{ lb.} = 29{\cdot}4 \text{ tons.}$$

230. *Example* II.—Required the greatest safe (**172**) load uniformly distributed on a rectangular beam of British oak projecting from a wall.

Length of beam = 6 ft. = 72 in. . . . (**200.**)
Breadth ,, = 6 in.
Depth ,, = 9 in.

Taking coef. (**172**) as 5; C (**203**) as 10,000. . . . (**231.**)

$$M = R. \quad . \quad . \quad . \quad (1, 196.)$$

$$M = \frac{w\,l^2}{2} \quad . \quad . \quad (9.) \qquad R = \frac{C\,a\,d}{6 \times Co} \quad . \quad . \quad . \quad (204.)$$

$$= (w\,l)\,36. \qquad\qquad = \frac{10{,}000 \times 54 \times 9}{6 \times 5}.$$

$$= 162{,}000$$

$$36\,(w\,l) = 162{,}000.$$

$$w\,l = 4{,}500 \text{ lbs.} = 2 \text{ tons.}$$

231. TABLE OF THE STRENGTH, &c., OF MATERIALS IN POUNDS AVOIRDUPOIS PER SQUARE INCH OF SECTION.

Materials.	*Ultimate Resistance to*				*Modulus of Elasticity.*	*Weight of a Cubic Foot in lbs.**
	Tension.	*Compression.*	*Shearing.*	*Cross Breaking, Modulus of Rupture.*		
Metals:						
Brass, Cast	18,000	10,300	...	...	9,000,000	487 to 524·4
,, Wire.........	49,000	...	...	...	14,230,000	533
Copper, Rolled.....	60,000	110,000	...	...	17,000,000	549
,, Bolts	36,000					556
Iron, Cast						
,, American best	40,000	110,000	27,700			
,, ,, average.....	16,000			40,000	18,000,000	444

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* The specific gravities may be readily found when it is known that a cubic foot of water weighs 62·5 lbs.

TABLE OF THE STRENGTH, ETC., OF MATERIALS—*continued.*

Materials.	*Ultimate Resistance to*				*Modulus of Elasticity.*	*Weight of a Cubic Foot in lbs.*
	Tension.	*Compression.*	*Shearing.*	*Cross Breaking, Modulus of Rupture.*		
METALS:—*cont.*						
Iron, Wrought .. ,, ,, Bar	54,000 to 72,000					
,, ,, ,, average	65,000	100,000*	50,000	...	29,000,000	480
,, ,, Plates	41,000 to 71,000	36,000 to 40,000	... ...	42,000 ...	25,000,000	
,, ,, ,, average	55,000					
,, ,, Joints—						
single riveted .	40,000					
double riveted †	52,000					
,, ,, Wire	84,000	...	...	...	15,000,000	
,, ,, ,, Cables ..	88,000					
Steel Bars	17,000 to 134,000	150,000	120,000	...	30,000,000	487 to 493
,, ,, average ..	95,000					
,, Plates........	80,000					
TIMBER:						
Ash.............	16,500	9,000	1,400	12,000 to 14,000	1,600,000	47
Beech............	11,500	9,000	...	9,000 to 12,000	1,350,000	43
Birch............	15,000	6,500	...	11,700	1,500,000	44·4
Box........... ..	20,000	10,000	...	...	2,000,000	60
Cedar............	11,400	5,800	...	7,400	486,000	30·4
Elm	12,000	10,000	1,400	6,000 to 9,700	1,000,000	34
Fir	12,000	5,500	500 to 1,500	5,000 to 10,000	700,000 to 2,000,000	30 to 44
Mahogany	8,000 to 20,000	8,000	...	7,600 to 11,500	1,600,000	35 to 53
Oak, English	10,000 to 18,000	10,000	2,300	10,000 to 13,600	900,000 to 1,700,000	43 to 62
,, American ..	12,000	6,000	...	10,600	2,000,000	54
Teak	10,000 to 15,000	12,000	...	12,000 to 19,000	2,300,000	41 to 61
HEMPEN CABLES	6,000					

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* It is difficult to estimate the compressive resistance of short blocks of wrought iron, as the material bulges very much under pressure.

† One and two rows of rivets (181). Joints of equal section to the plates taken through the line of rivets.—From numerous experiments by W. Fairbairn.

TABLE OF THE STRENGTH, ETC., OF MATERIALS—*continued.*

Materials.	*Ultimate Resistance to*				*Modulus of Elasticity.*	*Weight of Cubic Foot in lbs.*
	Tension.	*Compression.*	*Shearing.*	*Breaking, Modulus of Rupture.*		
STONES, CEMENT, ETC.:						
Brick, Fire	...	1,700				
„ Strong Red	275 to 300	1,100	...	...	...	125 to 135
„ Weak Red	...	550 to 800				
Chalk	...	400	...	...	...	117 to 174
Granite	...	11,000	...	...	...	164 to 172
Mortar, ord.	50	...	...	...	...	110
Limestone	...	4,500	...	...	2,500,000	170 to 180
Sandstone	...	2,000 to 5,500	...	1,100 to 2,360	...	130 to 157

232. VARIOUS METHODS OF DRAWING PARABOLAS, THE BASE AND HEIGHT BEING GIVEN.*

I. (Plate III., fig. 1.) BY ORDINATES OR OFFSETS FROM A TANGENT (E D) TO THE PARABOLA AT ITS VERTEX (D).

Through D draw D E parallel and equal to A C. The ordinates or offsets from any points in D E to the parabola will be proportional to the squares of the distances of those points from D. Thus, if the ordinate at *a* be 1, then the ordinate at *b*, *twice* the distance of *a* from D, must be 4; that at *c*, three times the distance, must be 9; and so on. To proceed practically: Divide E D into a number of equal parts (n) as at *a*, *b*, *c*, &c., fig. 1; then if E A be divided into (n^2) parts, each of these parts will be the required unit, 1 of which is the offset at *a*, 4 at *b*, 9 at *c*, and so on. Through the points *a′ b′ c′*, &c., thus determined, the required curve can be drawn.

II. (Plate III., fig. 2.) BY ORDINATES FROM THE BASE.

Divide the base (half of which is represented by A C) into an even number of equal parts; then if the height or ordinate at centre D C correspond to the square of half the number of those parts ($8 \times 8 = 64$), the

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* The terms height (or ordinate at centre) and base have been used instead of abscissa for the former, and double-ordinate for the latter, that the parabola might appear in a more simple light than perhaps it otherwise would have done. Any height can be adopted for the parabola; convenience for scaling off the moments (5), &c., being alone studied.

PLATE III.

CONSTRUCTION OF PARABOLAS.

Fig. 1.

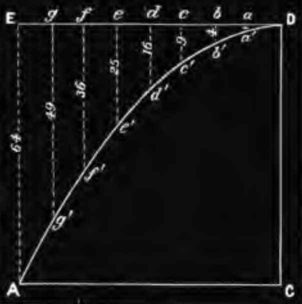


Fig. 2.

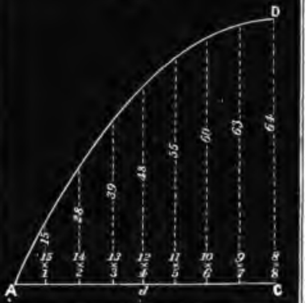


Fig. 3.

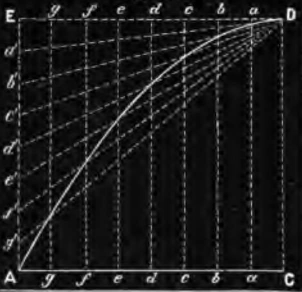


Fig. 4.

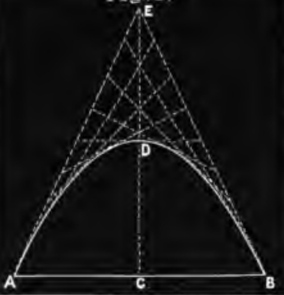


Fig. 5.

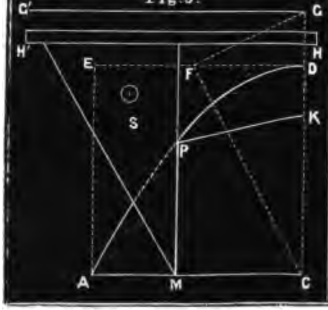
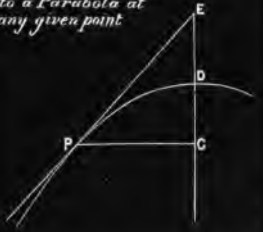


Fig. 6.

To draw a Tangent to a Parabola at any given point



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ordinate at any other point (*d* for instance) will be represented by the product of the numbers of parts in the two segments into which *d* divides the base (4 × 12 = 48). The parabola may then be drawn through the extremities of the ordinates.

III. (Plate III., fig. 3.) BY THE CONSTRUCTION OF A DIAGRAM.

Draw D E parallel and equal to A C; divide D E and E A similarly; the end E of E A corresponding to the end D of E D. Through *a*, *b*, &c., in E D, draw *a a*, *b b*, &c., parallel to D C. Join D to the several points *a′ b′*, &c., in E A. The parabola will pass through the intersection of *a a* with D *a′*, *b b* with D *b′*, &c.

NOTE. If this mode of construction be adopted when the ordinates are required at certain points only (and this will generally be the case in practice), *the actual curve need not be drawn, after the points it passes through have been determined.*

IV. (Plate III., fig. 4.) BY THE CONSTRUCTION OF A DIAGRAM.

On the base A B describe an isosceles triangle, whose height C E is double that of the required parabola. Divide the two sides A E, E B of the triangle into an even number of equal parts, and draw lines as in the figure. These lines will be tangents to the parabola, which may therefore be readily drawn.

V. (Plate III., fig. 5.) BY MEANS OF A STRING.

Draw E D equal and parallel to A C. Join C to F at the bisection of E D. Make C F G a right angle. Let F G intersect the production of C D. Make D K* = D G, parallel to F D; H H is a "straight-edge," against which slides the "set-square" S. A piece of thread or fine string equal in length to the distance A K is fixed, one end at K and the other at the point M, in the set square which will traverse the base A C as the set square slides along. A pencil, P, by which the string is kept tight, and close to the edge of the set square, will describe a true parabola.

TO DRAW A TANGENT TO A PARABOLA AT ANY POINT P. (Plate III., fig. 6.)

Draw P C perpendicular to the axis E C. Make D E = D C. Join E to P, and P E will be the required tangent.

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* K is the "focus" of the parabola. G G′ the directrix.

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